JOURNAL
OF
THE FRANKLIN INSTITUTE
DEVOTED TO SCIENCE AND THE MECHANIC ARTS

VOL. CLXXVII FEBRUARY, 1914 No. 2

EFFECT OF ELECTRICAL ENGINEERING ON MODERN INDUSTRY.*

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I.

THE use of electricity in modern civilized life is rapidly increasing: in lighting our homes, factories, streets; in industrial power applications; in domestic service, from the fan motor to the electric bell or the heating and cooking device; in transportation: while no great inroads have yet been made into the field of the steam locomotive, an entire system of electric railroads has sprung up all over the country, fully comparable in size and power demand with the steam railway system; large new industries have developed in electro-chemistry and electro-metallurgy, supplying us with materials unavailable before—as aluminum—or improving the production of other materials—as copper refining, etc.

All these applications are uses of *energy*. In nearly all, electrical energy is replacing some other form of energy used heretofore: chemical energy of fuel, or mechanical energy of steam or gas engines, etc.

* Presented at the meeting of the Electrical Section held Thursday, October 2, 1913.

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To understand the reasons which enable electrical energy to compete successfully with other forms of energy, which are longer and more familiarly known, we have to look into its characteristics.

Electrical energy can be transported—or, as we usually call it, transmitted—economically over practically any distance. Mechanical energy can be transmitted over a limited distance only, by belt or rope drive, by compressed air, etc.; heat energy may be carried from a central steam heating plant for some hundred feet with moderate efficiency, but there are only two forms of energy which can be transmitted over practically any distance,—that is, which in the distance of transmission are limited only by the economical consideration of a source of energy nearer at hand: electrical energy, and the chemical energy of fuel. These two forms of energy thus are the only competitors whenever energy is required at a place distant from any of Nature's stores of energy. Thus, when in the study of a problem of electric power transmission we consider whether it is more economical to transmit power electrically from the water power or the coal mine, or generate the power by a steam plant at the place of demand, both really are transmission problems, and the question is whether it is more economical to carry energy electrically over the transmission line, or to carry it chemically, as coal by the railroad train or boat, from the source of energy supply to the place of energy demand, where the energy is converted into the form required, as into mechanical energy by the electric motor or by steam boiler and engine or turbine.

Electrical energy and chemical energy both share the simplicity and economy of transmission or transportation, but electrical energy is vastly superior in the ease, simplicity, and efficiency of conversion into any other form of energy, while the conversion of the chemical energy of fuel into other forms of energy is difficult, requiring complicated plants and skilled attendants, and is so limited in efficiency as to make the chemical energy of fuel unavailable for all but very restricted uses: heating, and the big, high-power steam plant. Pressing the button turns on the electric light and thereby starts conversion into radiating energy: with chemical energy as source, either special fuels are required—in the candle, kerosene lamp—or a complex gas plant. Closing the switch starts the motor, whether a small fan motor, or a 1000-

horsepower motor supplying the water system of a city or driving the railroad train. With fuel as source of energy, boiler plant, steam engine, or turbine, with its numerous auxiliaries, with skilled attendants, etc., are necessary, and the efficiency is low except in very large units. To appreciate the complexity of the conversion of the chemical energy of fuel, compared with the simplicity of electrical energy conversion, imagine the domestic fan motor with coal as source of energy: a small steam engine, with boiler and furnace, attached to the fan: to start the fan, we have to make a coal fire and raise steam to drive the engine. This illustrates how utterly unavailable the chemical energy of fuel is for general energy distribution. General energy distribution, therefore, may justly be said to date from the introduction of electric power.

Equally true is the reverse: the conversion of mechanical or other energy into electrical is simple and economical, while the conversion into chemical energy is not. Hence, one of the two large sources of Nature's energy, the water power, was, before the days of electrical engineering, useless except to a very limited extent, since the location of the water power is rarely such that the energy could be used at its source. The water powers thus have really been made available only by the development of electrical transmission.

Characteristic of electrical energy is that it can be concentrated to an energy density higher than any other form of energy, and results can thus be produced by it which no other form of energy can bring about, or things done directly by the brute force of energy, as we may say, which formerly had to be brought about in a roundabout way.

Thus iron can be reduced from its ores by the chemical energy of coal in the blast furnace, but aluminum and calcium can not, as their chemical affinity is higher, and require the higher energy concentration available with electric power. Iron reduced in the blast furnace combines with carbon to cast iron. So calcium combines with carbon in the electric furnace to carbide, the starting material of acetylene, and of cyanamid and the modern fertilizer industry. Platinum can just be melted, and quartz softened, in the hottest flames of combustion: the oxy-acetylene flame and the oxy-hydrogen flame. But in the electric arc platinum and quartz and every existing substance, even tungsten and

carbon, can be melted and distilled or sublimed. Thus mighty industries have grown up and many new materials made available to man, as aluminum, silicon, calcium, chromium, the carbides, cyanamid, acetylene, etc.; others produced in a cheaper manner, as alkalies, hypochlorites, phosphorus, magnesium, sodium, etc.

Electricity as such is the most useless form of energy: it is not found in Nature in industrially available quantities, and finds no industrial use as electrical energy, but it is always produced from some other form of energy, and converted into some other form of energy: light, mechanical energy, chemical energy, heat, etc. That is, electrical energy is entirely the connecting link, the intermediary, by which energy is brought from the place where it is found to the place where it is used, or changed from the form in which it is found to the form in which it is used. Thus, on first sight, it appears a roundabout way, when, for instance, in modern electrical ship propulsion an electric generator is placed on the steam turbine, a motor on the ship propeller, a few feet away, though it is not different from practically every other use of electric energy: a transmission link, superior to any other transmission by the flexibility given by the simplicity and economy of conversion.

The most serious disadvantage of electrical energy is that it can not be stored. It is true, there exists the electric storage battery, and it is used to a large extent as stand-by battery in high-grade electric distribution systems to give absolute reliability of service, or as battery floating on a railway circuit to equalize fluctuations of power, or in special applications, as electric automobiles. It does not really store electrical energy, but stores energy by conversion of the electrical into chemical energy, and reconversion, in discharge, of the chemical into electrical energy.

The economic efficiency of the storage battery—using the term in the broad sense including interest on the plant investment and depreciation—is so low that the storage battery does not come into consideration in the industrial storage of energy,—that is, in making the rate of electrical energy consumption independent of that of energy production. We can best realize this by comparing electrical energy with the chemical energy of fuel: the latter can be stored with perfect economy. Thus, when using fuel as the source of energy—in a steam plant—no serious difficulty is met by the industry even if the fuel supply is interrupted

for months, as in the case of a supply by water, through the closing of the navigation by ice: we would simply bring in a sufficient coal supply to last until the navigation opens again in spring. But with electrical energy from a water power we could never dream of storing energy by storage battery to last over the two or three months during which the river runs dry and the water power fails.

This means that electrical energy must be consumed at the rate at which it is produced, and the cost of electrical energy thereby becomes dependent on the rate of the energy use. This is not the case with most other forms of energy, as, for instance, the chemical energy of fuel. The price of a ton of coal, as determined by the cost of supplying it, is the same whether I dump the coal into a furnace all at once, or whether I use it up at a uniform rate in a small stove, lasting for weeks. If I consume 2400 cubic feet of gas per day, its cost and thereby its price is the same whether I use the gas at a uniform rate throughout the day, of 100 cubic feet per hour, or whether I use the entire 2400 cubic feet in one hour, nothing in the remaining 23 hours: the gas is produced at whatever rate is most economical, stored in the gas holders and supplied from there at whatever rate it is required for consumption. If, however, I use 240 kilowatt-hours of electrical energy per day, it makes a very great difference in the cost of supplying this energy whether I use it at a uniform rate of 10 kilowatt-hours per hour, or whether I use the entire 240 kilowatt-hours in one hour, nothing in the remaining 23 hours. In the former case, 10 kilowatts of generating machinery are necessary in the steam or hydraulic station producing the electric energy, 10 kilowatts capacity in transmission lines, transformers, sub-station and distribution lines, to supply the demand. In the latter case, 240 kilowatts of generating machinery, 240 kilowatts of line and transformer capacity are absorbed, and that part of the cost of supplying the electric energy, which consists of interest in investment in the plant, of depreciation, etc.,—in short, the fixed cost,—is 24 times as high in the latter as in the former case. If the fixed cost approximates half the total cost in a steam plant, or is by far the largest part of the total cost in a hydraulic plant, it follows that in the case of concentrated energy used during a short time the cost of electric energy—and with it the price—will be very much larger—many times, possibly—as in the case of a uniform energy consumption.

Thus, due to the absence of storage, the cost of electrical energy essentially depends on the uniformity of the rate of its use,—that is, on the load factor, as the ratio of the average consumption to the maximum consumption.

If I use 240 kilowatt-hours of electrical energy in one hour, nothing during the remaining 23 hours, that part of the cost which is the fixed cost of plant investment and depreciation is 24 times as great as if I used the same amount of energy at a uniform rate throughout the day. In the former case, if somebody else uses 240 kilowatt-hours, but during another hour of the day, the same plant supplies his energy, and the fixed cost thus is cut practically in two,—that is, the cost of energy to both of us is materially reduced. Thus, again, the cost of electrical energy, and with it its price, depends on the overlap or not overlap of the use of the energy by different users, the so-called "diversity factor." The greater the diversity factor,—that is, the less the different uses overlap and the more their combination, therefore, increases the uniformity of the total energy demand, the "station load factor," —the lower is the energy cost. The cost of electrical energy for lighting, where all the demand comes during the same part of the day, is inherently much higher than the cost for uniform 24-hour service in chemical works, and with the increasing variety of load, with the combination of energy supply for all industrial and domestic purposes, the cost of energy decreases.

Thus, unlike other forms of energy, due to the absence of energy storage, electrical energy can have no definite cost of production, but, even supplied from the same generating station, its cost varies over a wide range, depending on the load factor of the individual use and the diversity factor of the different uses.

This feature, of necessity, must dominate the economical use of electrical energy in industrial, domestic, and transportation service.

II.

Civilization results in the complete interdependence of all members of society upon each other. Amongst the savages each individual, family, or tribe is independent, produces everything it requires. In the barbarian state some barter develops, followed by trade and commerce with increasing civilization. But up to a fair state of civilization—up to nearly a hundred years ago—all necessities of life were still produced in the immediate neighbor-

hood of the consumer, each group or territory still independent in its existence, and commerce dealing with such things only which were not absolutely necessary for life. All this has now changed, and in our necessities of life, as well as luxuries, we depend on a supply from distances of hundreds and thousands of miles: the whole world contributes in the supply of our food, clothing, building materials, etc.

That means, our existence is dependent on an efficient and reliable system of transportation and distribution of all needs of civilized life. Such has been developed during the last century in the system of steam railroads, which, in taking care of the transportation and distribution of commodities, have made modern civilization possible. For civilization means separation of production, in time and in location, from consumption, to secure maximum economy.

The necessities of civilized life consist of two groups: materials and energy. Our transportation system takes care of materials, but can not deal with the supply of energy, and the failure of an efficient energy supply has been and still is the most serious handicap which retards the advance of civilization. The transportation system could deal with the energy supply only in an indirect manner, by the supply of materials as carriers of energy, and when our railroads carry coal it is not the material which we need, but the energy which it carries. But this energy is available only to a very limited extent, as heat, and as mechanical power in big steam units; most of the energy demands of civilized life could not be satisfied by it. In any country village far away from the centres of civilization we have no difficulty to have delivered to us any material produced anywhere in the world; but even in the centres of civilization we could not get the energy to run a sewing machine or drive a fan without *electric power*. Thus, just as our steam railways and express companies take care of the transportation and distribution of materials, so civilization requires a system of transmission and distribution of energy, and our electric circuits are beginning to do this; and just as fifty to seventy-five years ago in the steam railroads, steamship lines, etc., the system of transportation and distribution of materials was developed, so we see all around us in the electric transmission systems the development of the system of the world's energy transmission in progress of development. When we see local

electric distribution systems combining, the big electric systems of our capital cities reaching out over the country, transmission lines interconnecting to networks covering many thousands of square miles, this is not merely the result of the higher economy of coöperation, of mass production, but it is the same process which took place in the steam railroad world some time ago, as a necessary requirement of coördination to carry out their function as carriers and distributers of materials in the case of the railroads, of energy in the case of the electric systems.

We must realize this progress, and the forces which lead to it, so as to understand what is going on, and to assist in the proper development, in avoiding, in the creation of the country's electrical network, whatever mistakes have been made in the development of the country's railway network.

Electricity, thus, is taking over the energy supply required by civilization as the only form of energy which, by its simplicity and economy of conversion, combined with economical transmission, is capable of supplying all the energy demands, from the smallest domestic need to the biggest powers. As we now begin to realize, the economic function of the steam engine is not the energy supply at the place of consumption, from the chemical energy of coal—it is too complicated and inefficient for this—but it is the conversion of chemical energy of coal into electrical energy in bulk, for transmission and distribution to the places of consumption.

If, then, electric power takes the place of steam power in our industries, etc., it is not merely the substitution of the electric motor for the steam engine or turbine. Such would rarely realize the best economy. The method of operation in all our industries, and especially those requiring considerable power, is largely—more than usually realized—determined by the characteristics of the power supply, and what is the most economical method with the steam engine as source of power may be very uneconomical with electric power supply, and electric power supply often permits a far more economical method of operation which was impossible with steam power. Thus the introduction of electricity as the medium of distributing the world's energy demand means a reorganization of our industrial methods, to adapt the same to the new form of power.

For instance, the steam engine requires skilled attendance,

and with its boiler plant, auxiliaries, etc., is a complex apparatus, is economical only in large units. Thus, when operating a factory or mill by steam power, one large engine is used, driving by shafts and counter-shafts, by pulleys and belts, and possibly wasting half or more of its energy in the mechanical transmission to the driven machines. But we could not economically place a steam engine at every one of the hundreds of machines in the factory. Substituting electrical power by replacing the engine by one large electric motor would be very uneconomical, as we can place a motor at every driven machine, and these small motors are practically as efficient—within very few per cent.—as one big motor would be, and all the belting and shafting, with its waste of energy, inconvenience, and danger, vanishes. With the steam engine as source of power, to run one or two machines only, to complete some work, requires keeping the big engine in operation, and therefore is extremely wasteful. With individual electric motors the economy is practically the same, whether only one or two motors are used, or the entire factory is in operation. On the other hand, with the steam engine, it makes no difference in the cost of power whether it is in operation from 8 A.M. to 6 P.M., or from 6 A.M. to 4 P.M. With electric power, in the former case the power demand would overlap with whatever lighting load the same supply circuit carries, but would not in the latter case, and the latter case thus would give a better load factor of the electric circuit, and thereby a lower cost of power. Again, with electric power, if very large power demands could be restricted to the periods of light load on the electric supply systems, this would reduce the cost of power. Nothing like this exists with the steam engine.

Electrical energy thus makes the power users economically more dependent upon each other, and thereby exerts a strong force toward industrial coördination,—that is, coöperation.

Another illustration of the industrial reorganization required to derive the full benefit of electric power is afforded by the traction problem. Very often a study of the electrification of a railway shows no economical advantage in the replacement of the steam locomotive by the electric locomotive, even when considering only passenger service. At the same time, an electric railway may parallel the same steam railway, offer better service at lower price, and show financially better returns than the steam

railway. But so, also, in the early days of steam, the steam engine in place of the horse in front of the stage coach was no success, and still the stage coach has gone and the steam locomotive has conquered; but it did not by replacing the horse, but by developing a system suited to the characteristics of the steam engine. The same repeats now in the relation of steam traction and electric traction. The steam engine is most economical in the largest units, and the economy of steam railway operation depends on the concentration of the load in as few and as large units as possible: therefore, the largest locomotive which can pass through bridges and around curves. Exactly the reverse is the condition of economy of electric traction: the economy depends on the distribution of the load as uniformly as possible in space and in time,—that is, small units at frequent intervals,—and therefore, while steam traction has gone to larger and larger units, in electric traction even the trailer car, so frequently used in the early days, has practically vanished. Obviously, then, the electric motor can not economically compete with the steam engine under the conditions of maximum economy of steam and minimum economy of electric operation, and electric traction under steam traction conditions shows marked economy only in the case of such heavy service that the maximum permissible train units follow each other at the shortest possible intervals,—that is, give maximum uniformity of load,—and thus the economic requirements of both forms of power coincide. These two instances may illustrate the changes in industrial operation which the introduction of electric power requires and which are taking place to-day.

To conclude, then: Electric energy is the only form which is economically suited for general energy transmission and distribution. Civilization depends on the supply of materials and of energy as its two necessities. The supply of materials is taken care of by the transportation system of the world. The supply of energy is being developed by the electrical transmission system, which with regard to energy becomes what the railway system is with regard to materials. Introduction of electric power in place of other forms of power rarely can be a mere substitution, but usually requires a change of the methods of power application, a reorganization of the industry, to secure maximum economy.

THE SKIN EFFECT AND ALTERNATING-CURRENT RESISTANCE.

BY

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WITH MATHEMATICAL TREATMENT BY

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I. SKIN EFFECT AND ALTERNATING-CURRENT RESISTANCE AS CLOSELY-RELATED PHENOMENA.

THE "skin effect" and alternating-current resistance (terms which we shall later define with care) are such closely-related phenomena that they cannot be adequately investigated, either mathematically or experimentally, the one apart from the other. We are therefore treating both these phenomena in the one paper.

In a conductor (maintained at a uniform temperature throughout its cross-section) which carries a direct current of uniform intensity, the current distributes itself with strictly uniform density over the cross-section of the conductor. This is the only distribution in which the rate of heat production is a minimum and it may be proved [1] that in the case of steady currents the distribution of the currents will always be such as to make the rate of heat production as small as possible. When, however, the impressed electromotive force becomes variable, alternating or oscillating, new forces arise which may change the current distribution over

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* To E. F. Northrup is due the initiation of the investigation herein recorded, the planning of the research, the experimental methods employed, and the physical treatment of the ideas and facts recorded; to Mr. John R. Carson, of the Department of Electrical Engineering of Princeton University, are wholly due the mathematical investigation and the writing of Section IV, with the curves exemplifying the formulæ obtained by him; Mr. Dudley Sanford and Mr. R. J. Kieffer, graduate students in electrical engineering in Princeton University, made all the experimental observations at the writer's request and under his immediate supervision, and prepared a report upon the work, which is in part here reproduced.

[1] "Elements of Electricity and Magnetism," paragraphs 194, 195, by J. J. Thomson.

the cross-section of the conductor in a way not at all consistent with the above principle. The phenomena of the current flow may assume great complexity. In certain cases of conductors of non-magnetic material, the current distribution over the cross-section of the conductor and the rate of heat production in the conductor can be calculated to an approximation which will meet any ordinary engineering requirements. In conductors of ferrous material, however, the phenomena assume a complexity beyond the power of any analysis to specify—except in so far as to reveal the general trend of the phenomena. The alternating-current resistance in such cases can only be determined precisely by experimental investigation.

The general effect of a variable impressed electromotive force is to cause a distribution of the resulting current such that the current density is greater near the circumference than near the axis of the conductor. As the entire cross-section of the conductor does not equally share in the carrying of the current, the total heat production in the conductor will not be the same as would be given by a uniform distribution of the current. Hence the alternating-current resistance is a different and larger quantity than the direct-current resistance.

While direct-current resistance is only a function of the nature of the material and temperature (when reduced to unit section and unit length), alternating-current resistance, on the other hand, is a function of many variables, and is especially complicated to consider in conductors of magnetic material. In the latter case it not only varies with the specific ohmic resistance of the material, the temperature, the section, and the frequency, but with the current and the permeability, this itself varying in a complex manner, over the cross-section of the conductor.

As this paper will show, the skin effect and the alternating-current resistance may assume large magnitudes in important cases which arise in engineering practice. It becomes, therefore, important to investigate, to measure, and to clearly describe (as best we may) these phenomena. As, in the writer's opinion, every mathematical investigation should be preceded by a physical consideration of the matter under investigation, we shall proceed in this way; and, following Mr. Carson's analysis, we shall give the experimental results which were obtained and some obvious deductions and suggestions to which one is naturally led.

II. CURRENT DISTRIBUTION; GENERAL STATEMENT AND ASSUMPTION.

In the case of a metallic conductor of circular cross-section, the statement that direct-current distributes itself uniformly over the cross-section is based upon the assumption that the temperature is uniform over the area of the section or upon the assumption that the material of the conductor is without a temperature coefficient. Now it is apparent that, if the current is sufficiently large to heat the conductor, the conductor losing heat from the surface will tend to become cooler at its surface than at its axis. This being so, the circumferential layers will have a lower resistivity than the axial portion, and hence the current will crowd toward the circumference, more so as the tendency to a temperature gradient from axis to circumference is increased. In the case of copper conductors of large circular cross-section made to carry direct currents which heat them considerably, a marked increase in current density at the circumference over the current density of the axis would undoubtedly occur. Thus an imitation of the skin effect may occur with direct currents. If the temperature of the surface of the conductor is registered with a thermocouple, and if this is called the temperature over the cross-section of the conductor, an apparent departure from Ohm's law would be observed in measuring the ohmic resistance, if the current used is such as to heat the conductor. In all mathematical treatments of direct-current flow such influences are ignored, though delicate measurements would surely reveal them.

When the current is alternating, the axial portion of the conductor is cut by a greater number of the expanding and contracting lines of induction than the circumferential portion. As a result of this, back electromotive forces are set up in the axial portion, and the current crowds towards the circumference, where it is least opposed by these back electromotive forces. If the temperature of the conductor is assumed uniform over its cross-section, and if the permeability of its substance is unity or is assumed constant, then it is possible to derive formulæ which precisely express the current density over the cross-section, the phase relations of the current elements in respect to each other at different points along the radius of the conductor, and the increase in the resistance of the conductor produced by the altered current

distribution. But for a precise expression or statement of what actually takes place neither of the above assumptions can be made. The current when crowded to the circumference (in the case of non-magnetic material with a temperature coefficient) will heat the circumferential portion. This heating will increase the resistance at this place and there will be a temperature gradient, too complicated to express, from axis to circumference which will modify the calculated current distribution. Of course, effects of this character must be small in the case of ordinary conductors used in power transmission, but in the case of the short, heavy conductors used for supplying large-capacity electric furnaces they may doubtless become considerable. In the case of large conductors of ferrous material, the assumption of a constant permeability from axis to circumference is so far from the truth that a mathematical analysis of the alternating-current resistance must and did entirely fail to predict phenomena of magnitude and real engineering importance. One important unpredicted phenomenon, which experiment revealed, is that the alternating-current resistance in ferrous materials reaches a sharp maximum with increasing current and then declines with an asymptotic curve (see curves, etc.).

We do not underate the value of analysis in dealing with the skin effect and alternating-current resistance, but wish attention drawn to the fact that the analysis can give no more truth than is put into the original assumptions—which must be made simple in character to make analysis possible. As a mathematical expression cannot give the whole truth, it is well to inquire if simple physical considerations may not yield information which, though also but a part of the truth, may nevertheless meet many of the engineering requirements besides giving clearness and vividness to the character and general trend of these very complex phenomena. We proceed to our conception of the physical aspect of the skin effect and alternating-current resistance.

III. PHYSICAL VIEW OF SKIN EFFECT AND ALTERNATING-CURRENT RESISTANCE.

Non-magnetic Material; Solid Conductor.

To simplify our considerations assume: That the conductor is circular in cross-section and very long; that the return con-

ductor is a hollow cylinder of infinite conductivity which closely fits the outgoing solid conductor, being insulated from it by very thin insulation; that its resistivity ρ is known and constant; that the frequency n and the total current I are invariable, and that the latter when alternating is of pure sine wave form. Assume further that the electrostatic field of force between the outgoing and return conductors in nowise affects the problem; that all calculations apply to a unit length of the conductor, and that over the cross-section of this unit length the impressed electromotive force per unit length of the conductor is strictly uniform.

The references are to Fig. 1, and the calculations are in C.G.S. units. Call G the radius of the conductor and g the distance measured along the radius from the axis to any point p within the conductor. When direct current of value I flows through the conductor the intensity of the magnetic force at any point p within the conductor is

$$T_g = \frac{2I_g}{G^2} \qquad (1)$$

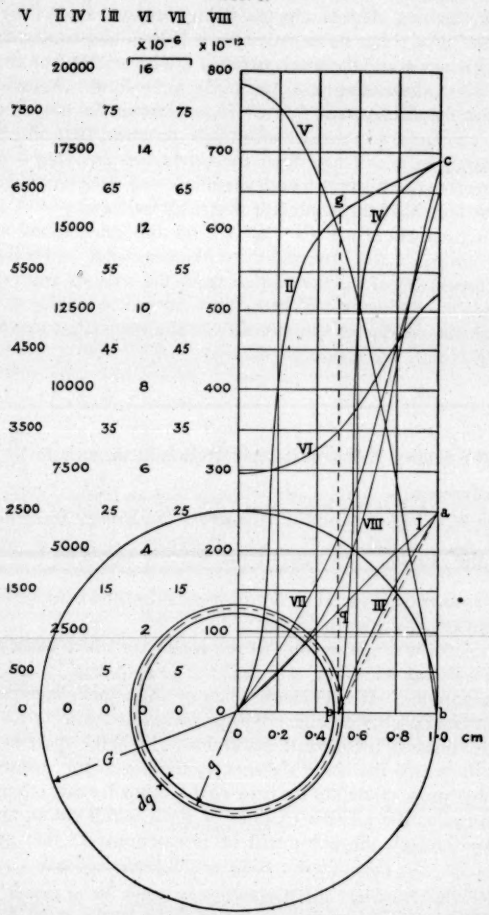
Thus the field increases uniformly from 0 at the axis to $\frac{2I}{G}$ at the circumference.

Curve I, Fig. 1, shows the intensity of the field at each point along the radius. The area of the triangle oab is $\frac{1}{2} G \times \frac{2I}{G} = I$, and is numerically equal to N, the total number of lines of force in the interior of the conductor.[2]

Consider a single annulus of the conductor which has a radius g and a radial width dg, and that the point p, Fig. 1, is located in this annulus. It is evident, when by the establishment of the current the number of lines of force increases from 0 to N, that as many lines of force cut this annulus as lie in the space between the point p and the circumference of the conductor; namely, by a number proportional to the area $pqab$. Thus for any other point on the radius the number of lines of force which cut an annulus passing through the point will be proportional to that part of

[2] Maxwell, "Electricity and Magnetism," paragraph 682, or consult "Some Newly Observed Manifestations of Forces in the Interior of an Electrical Conductor," by Edwin F. Northrup, *Phys. Rev.*, June, 1907.

FIG. 1.


V
II
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75
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a
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the area of the triangle *oab* which lies between the point and the circumference of the conductor. The expression for this portion of the area of the triangle is

$$N_p = \frac{G^2 - g^2}{G^2} = I \quad \ldots\ldots (2)$$

where I is the maximum value of the total current.

If the current, which gives rise to the lines of force, alternates as a sine wave with a frequency n, the annulus will be cut by N_p lines of force four times in each complete cycle: once with expanding lines of force and once with contracting lines of force, the current having a positive direction, and once with expanding and once with contracting lines of force, the current having a negative direction. Hence an electromotive force will be induced in the annulus parallel to the axis of the conductor, and the average value of this electromotive force will be:

$$E_{avg} = \frac{4n\,(G^2 - g^2)}{G^2} I \quad \ldots\ldots (3)$$

Since the maximum value of the current is $\frac{\pi}{2}$ times the average value I_{avg},

$$E_{avg} = \frac{2\pi n\,(G^2 - g^2)}{G^2} I_{avg} \quad \ldots\ldots (4)$$

and since the average value of the electromotive force bears the same relation to its effective value [namely,

$$E_{avg} = \frac{2\sqrt{2}}{\pi} E_{eff}]$$

as the average value of the current bears to its effective value, we have,

$$E = \frac{2\pi n\,(G^2 - g^2)}{G^2} I \quad \ldots\ldots (5)$$

where now we are to understand by E and I effective values of electromotive force and current, both being in C.G.S. electromagnetic units.

If we call $I_d = \frac{I}{\pi G^2}$, the current density of the effective

current *on the temporary assumption that the current is uniformly distributed over the cross-section of the conductor,* the value of the effective induced electromotive force becomes:

$$E_b = 2\pi^2 n (G^2 - g^2) I_d \quad \ldots\ldots (6)$$

By E_b we are to understand the induced electromotive force per unit length of the conductor at all points distant g from the axis, and which in general will differ in value and phase from both the impressed electromotive force and the electromotive force consumed by ohmic resistance. The electromotive force consumed by ohmic resistance will be equal to the product of the resistivity ρ of the material and the current density, or

$$E_f = \rho I_d \quad \ldots\ldots (7)$$

This electromotive force will have the same phase as the current, and it will be in quadrature with the induced electromotive force at any annulus of radius g. The impressed electromotive force, E_i (which is uniform in magnitude and phase over the cross-section of the conductor), is the vectorial sum of E_f and E_b, or

$$E_i = \sqrt{E_f^2 + E_b^2} \quad \ldots\ldots (8)$$

From equations (6), (7), and (8) we obtain,

$$I_d = \frac{E_i}{[\rho^2 + 4\pi^4 n^2 (G^2 - g^2)^2)]^{\frac{1}{2}}} \quad \ldots\ldots (9)$$

The direct-current density when the direct impressed electromotive force is E_i is

$$I'_d = \frac{E_i}{\rho} \quad \ldots\ldots (10)$$

The ratio

$$\frac{I'_d}{I_d} = \sqrt{1 + \frac{4\pi^4 n^2}{\rho^2} (G^2 - g^2)^2} \quad \ldots\ldots (11)$$

With the assumption made that the distribution of the lines of force over the cross-section of the conductor is that which would exist if the current were uniformly distributed over the cross-section, equations (5), (6), (9), and (11) are true equa-

tions. In Fig. 1, curve V is a plot of equation (6) where $G = 1$ cm., $n = 100$ cycles, and $I_d = \frac{12.5}{\pi} = 3.979$. (Note that this curve has the same form as the curve which expresses the hydrostatic pressure in the interior of a conductor carrying a uniformly distributed current.)[3]

Curve VI is a plot of equation (9) in which the values chosen for the constants are, $E_i = 1$, $\rho = 96 \times 10^3$ (the approximate specific resistance of mercury in C.G.S. units), $n = 7072$ cycles, and $G = 1$ cm. By the nature of the assumption made above, curves V and VI can only give an approximation to the truth. As curve VI shows, the current distributes itself more densely toward the circumference, and the magnetic field is no longer distributed as it is with uniform current distribution in the manner given by curve I. If all the current were flowing in that portion of the conductor which forms a cylinder of internal radius d and external radius G, then the distribution of the magnetic field may be obtained from the equation,

$$T_c = 2I \frac{g^2 - d^2}{(G^2 - d^2)\, g} \quad [4] \qquad \ldots\ldots (12)$$

Calling $d = \frac{G}{2}$, we obtain

$$T_c = 2I \frac{4g^2 - G^2}{3\, G^2 g} \qquad \ldots\ldots (13)$$

Curve III is a plot of equation (13) in which $G = 1$ and $I = 12.5$. Here the area of the figure *pab* gives the total number of lines of force in the cylinder when the current in the cylinder is I. It will be noted that these lines are very much less in number than the lines N_r [equation (2)]. Hence it follows that when the skin effect has begun to manifest itself and the current is crowded toward the circumference, the tendency to a further crowding of the current toward the circumference is diminished.

[3] "Some Newly Observed Manifestations of Forces in the Interior of an Electric Conductor," by Edwin F. Northrup, *Phys. Rev.*, June, 1907, p. 487, Eq. (6).

[4] *Ibid.*, p. 479, Eq. (3).

We conclude, therefore, that the true curve which expresses the current density at different points along the radius of the conductor must be flatter, and never steeper, than the curve obtained in the manner of curve VI drawn upon the assumption of a field of force disposed as given by uniform current distribution.

In curve VI, the ratio of the current density at the circumference to the current density at the axis is $\frac{10.43}{5.92} = 1.760$. It is shown by the analysis in Section IV that the true ratio of the current density at the circumference to the current density at the axis, when the same constants are used, is 1.50.

It is now possible to calculate the ratio of the alternating-current resistance to the direct-current resistance with the aid of graphical constructions and to obtain a fairly close approximation to the true value of this ratio. By definition the alternating-current resistance is

$$R_{ac} = \frac{\text{total watts developed per unit length}}{\text{square of total current.}},$$

and the direct current resistance is $R_{dc} = \frac{\rho}{\pi G^2}$.

Let $R_{ac} = \frac{W}{I_t^2}$, where W = total watts per unit length and I_t = the total current.

$$\frac{R_{ac}}{R_{dc}} = \frac{\pi G^2}{\rho} \cdot \frac{W}{I_t^2}.$$ Let the conductor be of unit radius.

Then $G = 1$ and $\frac{R_{ac}}{R_{dc}} = \frac{\pi}{\rho} \cdot \frac{W}{I_t^2}$. .....(14)

(*a*) To obtain I_t^2.

Let the impressed electromotive force per unit length = 1.
Then by Equation (9) when $G = 1$,

$$I_d = \frac{1}{\sqrt{\rho^2 + 4\pi^4 n^2 (1-g^2)^2}} .....(15)$$

If we plot the curve for I_d, letting $\rho = 96 \times 10^3$ and $n = 7072$, we obtain curve VI.

If all the ordinates of curve VI are multiplied by $2\pi g$, curve VII is obtained. If any ordinate of this curve is multiplied by dg, we obtain the current which is flowing in an annulus of radius

g and radial width dg. The current, however, in each annulus will have a different phase. The phase of the current, in respect to the impressed electromotive force, flowing in any annulus of radius g is given by the relation,

$$\text{Tan}\,\omega = \frac{2\pi^2(1-g^2)n}{\rho} \qquad \ldots\ldots(16)$$

Thus the phase of the current changes continuously from the axis to the circumference, being tan $\omega = \frac{2\pi^2 n}{\rho}$ at the axis and tan $\omega = 0$ at the circumference. With the constants and the method of calculation we have chosen, the current at the axis lags behind the impressed electromotive force 55° 29′. The actual angle of lag (using the same constants) as obtained by the analytical method of Section IV is 70°. Since the currents which flow in the annuli of different radii have different phases, the value of the total current cannot be obtained by algebraic addition. Hence the area of curve VII does not give the total current. The addition of the currents in all the annuli must be made vectorially. This vectorial addition may be made as follows:

Find values of tan $\omega = \frac{2\pi^2 n}{\rho}(G^2-g^2)$ when g is varied from 0 to G. As we have taken $G = 1$, values of the tangent may be found for eleven values, as 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. From a table of tangents obtain the corresponding values of ω. From a table of cosines find the corresponding values of cos ω, and from a table of sines find the corresponding values of sin ω. Multiply the ordinates of a curve corresponding to curve VII by the cosine values. The ordinates when so multiplied give the components of the current in phase with the electromotive force consumed by ohmic resistance. Hence if we plot a new curve in which the ordinates of curve VII $\times$ cos ω are ordinates and values of g are abscissæ, the area which lies between this curve and the horizontal axis is equivalent to the integral

$$A_{\cos\omega} = 2\pi \int_0^{G=1} I_d \cos\omega\, g \cdot dg$$

$A_{\cos\omega}$ is the component of the total current in phase with the electromotive force consumed by ohmic resistance. The area of

this curve is readily obtained by adding 11 evenly-spaced ordinates and dividing by 11, and multiplying by G, in this case by unity.

Next plot a curve where the ordinates of curve VII sin ω are made ordinates and values of g are made abscissæ. The area which lies between this curve and the horizontal axis is equivalent to the integral,

$$A_{\sin\omega} = 2\pi \int_0^{G=1} I_d \sin\omega \, g\,dg$$

and is equal to the component of the total current which is in quadrature with the electromotive force consumed by ohmic resistance.

The square of the total current is now

$$I_t^2 = A_{\sin\omega}^2 + A_{\cos\omega}^2 \qquad (17)$$

In performing these operations it is not necessary to actually plot any of the curves. One may operate upon the numerical values as if they were actually plotted. These operations when performed for the illustrative example in which $G=1$, $n=7072$, and $\rho = 96 \times 10^8$ give

$$A_{\sin\omega}^2 = 127.7 \times 10^{-12},$$

$$A_{\cos\omega}^2 = 532.2 \times 10^{-12}; \text{ and hence}$$

$$I_t^2 = 659.9 \times 10^{-12}.$$

(b) To obtain W:

If all the ordinates of curve VI are squared and multiplied by ρ, we obtain the watts per cubic centimetre at each point along the radius. If after these ordinates are squared and multiplied by ρ we then multiply each product by $2\pi g$, we shall obtain a curve the area between which and the horizontal axis is equal to W.

Let $B' = W$ be the area so obtained. If all the ordinates of curve VI are squared and then multiplied by $2\pi g$ only, we shall obtain a curve the area between which and the horizontal axis will be B where $B' = B\rho$. Curve VIII, Fig. 1, is such a curve.

We now obtain for the ratio of the alternating-current to the direct-current resistance

$$\frac{R_{ac}}{R_{dc}} = \frac{W}{I_t^2} \cdot \frac{\pi}{\rho} = \frac{\pi B}{I_t^2} = \frac{\pi B}{A_{\sin \omega}^2 + A_{\cos \omega}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

In the illustrative example, the value found for B is 229×10^{-12}, and hence for this particular case

$$\frac{R_{ac}}{R_{dc}} = \frac{3 \cdot 1416 \times 229 \times 10^{-12}}{659 \cdot 9 \times 10^{-12}} = 1.090.$$

This value is too small by 6 per cent., the true value being 1.160.

The explanation of the discrepancy is interesting. The graphical method of calculation is based upon our original assumption that the field of magnetic force has the distribution in the interior of the conductor which would result from current uniformly distributed over the cross-section of the conductor. The effect of this assumption is to give a greater ratio of the current density at the circumference to the current density at the axis than would actually exist. It will also cause the calculated values, at different points along the radius, of the angle of lag of the current to be too small. This will be understood if we note that the angle of lag of the current at the axis behind the current at the circumference is determined not by the actual number of lines of force which cut any two annuli lying between axis and circumference, but by the proportionate number which cuts the two annuli. Now if the field distribution is that due to uniform current distribution, a smaller percentage of lines will cut the inside annulus as compared with those which cut the outside annulus than will be the case if the field is that due to current having an increasing density toward the circumference.

Again, if we note that the tangent of a phase angle is proportional to self-induction divided by resistance, we have $\tan \omega \propto \frac{L}{\rho}$

Also, $L \propto \frac{\text{flux}}{I_d}$. Now we have assumed I_d, the current density at the axis greater than it really is after the skin effect has begun

to manifest itself. This assumption makes L and hence $\tan \omega$ too small. As stated above, at the axis we found $\omega = 55° 29'$, whereas it is 70°.

Since in calculating the value of the total current we take products of current densities (which are too large) by cosines and sines of phase angles (which are too small), the errors, due to the nature of the fundamental assumption made, partially neutralize. As a result we find the ratio $\frac{R_{ac}}{R_{dc}}$ too small, but only by 6 per cent.

We may calculate the ratio $\frac{R_{ac}}{R_{dc}}$ by taking the ratio of the watts being consumed for any instantaneous value of the current to the square of the integral value of the instantaneous current in phase with the electromotive force consumed by ohmic resistance. To perform this calculation we proceed as follows: Multiply all the ordinates of curve VI for the current density by the cosine of the phase angle. Square these values and multiply by $2\pi\rho g$. Plot the resulting curve and take its area. This area will equal the total watt consumption at a given instant. Let this instant be when the current has zero phase at the circumference. Now find, by the procedure given above, the component of the total current in phase with the electromotive force consumed by resistance. Square this component. Divide the first result by the second and multiply by $\frac{\pi}{\rho}$, the reciprocal of the direct-current resistance. The result is the required ratio.

Expressed analytically:

$$\frac{R_{ac}}{R_{dc}} = \frac{2\pi\rho \int_0^{G=1} (I_d \cos\omega)^2 \, g\,dg}{\left[2\pi \int_0^{G=1} I_d \cos\omega \; g\,dg\right]^2} \frac{\pi}{\rho}$$

or

$$\frac{R_{ac}}{R_{dc}} = \frac{\int_0^{G=1} (I_d \cos\omega)^2 \, g\,dg}{\left[2 \int_0^{G=1} I_d \cos\omega \; g\,dg\right]^2} \qquad (19)$$

This calculation was made for the illustrative example chosen and the result was:

$$\frac{R_{ac}}{R_{dc}} = \frac{581 \times 10^{-12}}{532.2 \times 10^{-12}} = 1.092,$$

which is the result obtained by the first method within the error of the numerical calculations.

Magnetic Material; Solid Conductor.

If the conductor consists of iron, the phenomena are profoundly modified both in magnitude and character. The lines in the interior of the conductor are now lines of induction instead of lines of force, and, due to the large permeability of iron, these lines are very numerous. They are not disposed in any way which can be expressed by an equation, because the permeability μ varies with the intensity of the field, which itself varies from axis to circumference. At any point distant g from the axis of a solid conductor the induction per square centimetre will be

$$B = \mu T_g = \frac{2\mu Ig}{C^2} \quad \ldots\ldots\ldots\

which are equal in number to the area *pgcb*. The tendency for alternating current to crowd toward the circumference of the conductor is increased when the total number of lines of induction which cut any annulus in the interior of the conductor is increased. Now when the total current in the iron conductor is increased the flattening of the curve of induction begins nearer the axis, and hence the proportionate amount of the current which is crowded toward the circumference is decreased. The result is that the alternating-current resistance, instead of continually increasing with increase in current, reaches a maximum and then begins to decrease. For very large total current nearly all the iron of the conductor would approach magnetic saturation and then the skin effect would partake of the same magnitude and character as it would in a conductor of non-magnetic material.

That these conclusions are entirely justified is shown by the experimentally obtained curves given later on.

Curve IV has been drawn to show how the lines of induction would be disposed if the same current I had all become crowded into the portion of the conductor which lies beyond the middle point of its radius. Not only is the curve much steeper than curve II, but the total number of lines equal to the area included between it and the horizontal axis is considerably less than the total number which is equal to the area included between the portion *pgcb* of curve II and the axis. It is concluded that any formulæ deduced to express the ratio of the alternating-current resistance to the direct-current resistance, when it includes μ assumed constant must be for iron conductors only roughly approximate and cannot have much value. We must, therefore, resort to experiment to determine this ratio.

In any method yet devised for measuring the alternating-current resistance it is necessary to compare the alternating-current resistance of the sample with the alternating-current resistance of a standard and to assume that the alternating-current resistance of this standard is either the same as its direct-current resistance or that its departure from its direct-current resistance can be accurately calculated. Experience and calculation have shown that mercury, because of its small temperature coefficient, large specific resistance, and because of the ease with which the resistance of a column of the liquid may be varied, is the best material to use for a standard.

In the mathematical section of this paper, four curves have been drawn which accurately express the ratio of the alternating current to the direct-current resistance of a column of mercury of circular cross-section. Referring (Fig. 10) to the curve for 60 cycles, it will be observed that this ratio for a column of 10 cm. in diameter is only 1.008 and for one 6 cm. in diameter only 1.001. Hence a column of mercury of this latter diameter could be used for a standard at 60 cycles without correction with an error of only 0.1 of 1 per cent.

From the above physical discussion of the skin effect it plainly appears that any exact mathematical expression for the ratio of the alternating-current resistance to the direct-current resistance must assume the form of an infinite series. We now proceed to the deduction of exact mathematical expressions for this ratio in the case of conductors of circular cross-section.

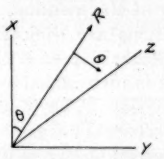
IV. MATHEMATICAL INVESTIGATION OF SKIN EFFECT AND ALTERNATING-CURRENT RESISTANCE.

In the present investigations we shall start with Maxwell's Equations, the basis of modern electromagnetic theory. These equations are the mathematical formulation of two laws:

(1) The live integral of the magnetic force around any closed curve is equal to 4π times the total current flowing through the closed curve.

(2) The live integral of the electric force around any closed curve is equal to the time rate of diminution of the total magnetic induction included by the closed curve.

FIG. 2.



Since we are dealing with electromagnetic effects, propagated along a cylindrical conductor, we shall find it convenient to write our equations in cylindrical coördinates, referred to the axes R,

θ, and Z. Let the Z axis coincide with the axis of the cylindrical conductor. Then the R and θ axes are in the plane perpendicular of the Z axis; the R axis is radial and the θ axis is tangential, as shown in Fig. 2.

We shall denote the component electric and magnetic forces E and H along the axes Z, R, and θ by the subscripts, z, r, and θ. We shall denote the distance along the vector R from the axis Z by r.

The mathematical formulation of the first law is now:

$$\left.\begin{aligned}
\left(4\pi\lambda + K\frac{d}{dt}\right)E_z &= \frac{1}{r}\frac{\delta^*}{\delta z}(rH_\theta) - \frac{1}{r}\frac{\delta}{\delta\theta}(H_R)\\
\left(4\pi\lambda + K\frac{d}{dt}\right)E_R &= \frac{1}{r}\frac{\delta}{\delta\theta}(H_z) - \frac{\delta}{\delta z}(H_\theta)\\
\left(4\pi\lambda + K\frac{d}{dt}\right)E_\theta &= \frac{\delta}{\delta z}(H_R) - \frac{\delta}{\delta r}(H_z)
\end{aligned}\right\} \text{(I)}$$

λ is the conductivity and K the specific inductive capacity of the medium. All quantities are assumed as expressed in electromagnetic C.G.S. units.

The formulation of the second law gives the three equations:

$$\left.\begin{aligned}
-\mu\frac{d}{dt}H_Z &= \frac{1}{r}\frac{\delta}{\delta r}(rE_\theta) - \frac{1}{r}\frac{\delta}{\delta\theta}E_R\\
-\mu\frac{d}{dt}H_R &= \frac{1}{r}\frac{\delta}{\delta\theta}(E_z) - \frac{\delta}{\delta z}E_\theta\\
-\mu\frac{d}{dt}H_\theta &= \frac{\delta}{\delta z}E_R - \frac{\delta}{\delta r}E_z
\end{aligned}\right\} \text{(II)}$$

These equations may be verified by transformation of Maxwell's Equations.

μ is the permeability of the medium.

These general equations are difficult to handle. We shall therefore simplify our work by introducing the assumption that the cylindrical conductor is surrounded by a concentric cylindrical shell of conducting material. The inner and outer conductors are separated by the dielectric. This assumption makes everything symmetrical in planes perpendicular to the axis of Z, and it follows that $E_\theta = 0$ and $\frac{\delta}{\delta\theta} = 0$.

* Partial differentiation.

The six equations, I and II, now reduce to:

$$\left(4\pi\lambda + K\frac{d}{dt}\right)E_z = \frac{1}{r}\frac{\delta}{\delta r}(rH_\theta)$$

$$\left(4\pi\lambda + K\frac{d}{dt}\right)E_R = -\frac{\delta}{\delta z}(H_\theta)$$

$$-\mu\frac{d}{dt}H_\theta = \frac{\delta}{\delta z}E_R - \frac{\delta}{\delta r}E_z$$

$$E_\theta = H_R = H_z = 0.$$

That is, the electric force has only two components, one radial and one axial, while there is but one component of magnetic force which is tangential to circles in the $R\ \theta$ plane, centred on Z axis. Since there is but one component of magnetic force we shall drop the subscript and denote it by H.

We therefore get, writing H for H_θ,

$$\left(4\pi\lambda + K\frac{d}{dt}\right)E_z = \frac{1}{r}\frac{\delta}{\delta r}(rH) \qquad (a)$$

$$\left(4\pi\lambda + K\frac{d}{dt}\right)E_R = -\frac{\delta}{\delta z}(H) \qquad (b)$$

$$-\mu\frac{d}{dt}H = \frac{\delta}{\delta z}E_R - \frac{\delta}{\delta r}E_z \qquad (c)$$

The foregoing equations express the electric and magnetic force as functions of Z, r, and the time t. If a steady state has been reached, we can, with complete generality, assume that these quantities are harmonic functions of time. Now, instead of taking them as sine and cosine functions of time, let us assume that E_R, E_z and H contain the factor ε^{ipt} where $i = \sqrt{-1}$. Now since

$$\varepsilon^{ipt} = \cos pt + i \sin pt,$$

the expressions for the electric and magnetic forces will be complex quantities. In the final result we ignore the imaginary part and take the real part of the expression as the solution. This is a well-known device in the treatment of differential equations, and enables us to get rid of the explicit appearance of the time in our equations.

If E_R, E_z and H contain the factor ε^{ipt}, then

$$\frac{d}{dt} = ip, \frac{d^2}{dt^2} = -p^2$$

We therefore get

$$(4\pi\lambda + Kip)\, E_z = \frac{1}{r}\frac{\delta}{\delta r}(rH) \tag{1}$$

$$(4\pi\lambda + Kip)\, E_R = -\frac{\delta}{\delta z} H \tag{2}$$

$$-\mu ipH = \frac{\delta}{\delta z} E_R = \frac{\delta}{\delta r} E_z \tag{3}$$

We have here three simultaneous equations involving E_z, E_R and H. We have to solve for E_z, E_R and H.

Multiplying (1) by μip, we get

$$(4\pi\lambda\mu ip) - Kp^2)\, E_z = \mu ip \frac{\delta}{\delta r} H + \mu ip \frac{H}{r}. \tag{4}$$

From (3) we have

$$\mu ip\, H = \frac{\delta}{\delta r} E_z - \frac{\delta}{\delta z} E_R$$

$$\mu ip \frac{\delta H}{\delta r} = \frac{\delta^2}{\delta r^2} E_z - \frac{\delta}{\delta r \delta z} E_R \tag{5}$$

$$\mu ip \frac{H}{r} = \frac{1}{r}\frac{\delta}{\delta r} E_z - \frac{1}{r}\frac{\delta}{\delta z} E_R \tag{6}$$

Substituting (5) and (6) in (4) we get

$$(4\pi\lambda\mu ip - K^2p)\, E_z = \frac{\delta^2}{\delta r^2} E_z + \frac{1}{r}\frac{\delta}{\delta r} E_z - \frac{\delta}{\delta z}\left\{\frac{\delta}{\delta r} E_R + \frac{1}{r} E_R\right\} \tag{7}$$

But by Laplace's equations in cylindrical coördinates

$$\frac{\delta}{\delta r} E_R + \frac{1}{r} E_R + \frac{\delta}{\delta z} E_z = 0$$

in a continuous medium.

Equation (7) then becomes

$$(4\pi\delta\mu ip - K\mu p^2)\, E_z = \frac{\delta^2}{\delta z^2} E_z + \frac{\delta^2}{\delta r^2} E_z + \frac{1}{r}\frac{\delta}{\delta r} E_z \tag{8}$$

Writing

$$(4\pi\lambda\mu i\rho - K\mu\rho_3) = a^2,$$

$$a^2E_z = \frac{\delta^2}{\delta z^2}E_z + \frac{\delta^2}{\delta r^2}E_z + \frac{1}{r}\frac{\delta}{\delta r}E_z \tag{9}$$

By exactly analogous methods we get

$$a^2E_R = \frac{\delta^2}{\delta z^2}E_R + \frac{\delta^2}{\delta r^2}E_R + \frac{1}{r}\frac{\delta}{\delta r}E_R - \frac{E_R}{r^2} \tag{10}$$

Since H is determined by equation (3), if E_z and E_R are known, we shall not solve for H.

We shall solve equation (9) for E_z first and then find the solution of (10) for E_R. Equation (9) is a partial differential equation in r and z. In searching for a solution we shall assume that it is of the form

$$E_z = R \cdot Z$$

when R is a function of r alone, and Z of z alone. Substituting in equation (9) RZ for E_z we get

$$a^2R \cdot Z = R\frac{\delta^2Z}{\delta z^2} + Z\frac{\delta^2R}{\delta r^2} + \frac{Z}{r}\frac{\delta R}{\delta r}, \text{ or}$$

$$\frac{1}{Z}\frac{\delta^2Z}{\delta z^2} + \frac{1}{R}\frac{\delta^2R}{\delta r^2} + \frac{1}{rR}\frac{\delta R}{\delta r} = a^2 = \text{ constant.}$$

Since the first term can by hypothesis be a function of z alone and the second and third of r alone, we must have

$$\frac{1}{Z}\frac{\delta^2Z}{\delta z^2} = a^2 - m^2 \tag{11}$$

$$\frac{1}{R}\frac{\delta^2R}{\delta r^2} + \frac{1}{rR}\frac{\delta R}{\delta r} = m^2 \tag{12}$$

where m is an unknown constant

Writing (11)

$$\frac{1}{Z}\frac{\delta^2Z}{\delta z^2} = -\beta^2 \quad \text{when} \quad \beta^2 = m^2 - a^2$$

we have as a solution

$$Z = A_1\epsilon^{i\beta z} + A_2\epsilon^{-i\beta z} \tag{13}$$

where A_1 and A_2 are constants of integration.

Equation (12) cannot be integrated as it stands, but by a simple transformation of the independent variable it can be reduced to a standard form. Writing $x = imr$ (12) becomes

$$\frac{\partial^2 R}{\partial x^2} + \frac{1}{x}\frac{\delta R}{\delta x} + R = 0. \tag{14}$$

Equation (14) is a particular case of the general differential equation

$$\frac{\partial^2 R}{\partial x^2} + \frac{1}{x}\frac{\delta R}{\delta x} + \left(1 - \frac{n^2}{x^2}\right) R = 0.$$

The solution of this general equation is a highly convergent infinite series, known as Bessel's Function of the n^{th} order, and denoted by $J_n(x)$. The only two orders we shall have to deal with in the present investigation are the zeroth and the first. These are

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \frac{x^8}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8^2} \cdots\cdots\cdots$$

$$J_1(x) = \frac{x}{2}\left\{1 - \frac{x^2}{8} + \frac{x^4}{8 \cdot 24} - \frac{x^6}{8 \cdot 24 \cdot 48} \cdots\cdots\cdots\right\}$$

From Equation (14), we have then

$$R = J_0(x) = J_0(imr) \tag{15}$$

$E_z = R_1 Z$, and we have as our expression for E_z

$$E_z = \left\{A_1 \varepsilon^{i\beta z} + A_2 \varepsilon^{-i\beta z}\right\} J_0(imr) \cdot \varepsilon^{ipt} \tag{16}$$

The factor ε^{ipt} is introduced in accordance with our original assumption that all variables contained the factor ε^{ipt}.

Equation (10) for E_R is solved by exactly analogous methods, giving as a solution

$$E_R = \left\{B_1 \varepsilon^{i\beta z} + B_2 \varepsilon^{-i\beta z}\right\} J_1(imr) \varepsilon^{ipt} \tag{17}$$

The relations between the constants of integration in the expressions for E_z and E_R are obtained by recourse to Laplace's equation as given above:

$$\frac{\delta}{\delta z} E_z + \frac{\delta}{\delta r} E_R + \frac{1}{r} E_R = 0$$

If we substitute in the above for E_z and E_R the values given by (16) and (17), and remember that

$$\frac{\delta}{\delta x} J_1(x) = J_0(x) - \frac{1}{x} J_1(x),$$

we get

$$\left\{ i\beta A_1 \varepsilon^{i\beta z} - i\beta A_2 \varepsilon^{-i\beta z} \right\} J_0(i\omega r) + (im) \left\{ B_1 \varepsilon^{i\beta z} + B_2 \varepsilon^{-i\beta z} \right\} J_0(imr) = 0.$$

whence

$$\varepsilon^{i\beta z} \left\{ i\beta A_1 + imB_1 \right\} + \varepsilon^{-i\beta z} \left\{ -iBA_2 + imB_2 \right\} = 0$$

whence

$$B_1 = \frac{-\beta}{m} A_1$$

$$B_2 = +\frac{\beta}{m} A_2.$$

We therefore have

$$E_z = J_0(imr) \left\{ A_1 \varepsilon^{i\beta z} + A_2 \varepsilon^{-i\beta z} \right\} \varepsilon^{ipt} \tag{16}$$

$$E_R = \frac{-\beta}{m} \left[J_1(imr) \right] \left\{ A_1 \varepsilon^{i\beta z} - A_1 \varepsilon^{-i\beta z} \right\} \varepsilon^{ipt} \tag{18}$$

$$\text{when } im = i\sqrt{a^2 + \beta^2}$$

A complete solution of the problem involves a determination of the exponential constant β which gives the attenuation and phase change of current and electric force along the conductor. β is a function of the dimensions and electrical constants of the inner and outer conductors. For our purpose, however, which is to study the distributions of the current in the inner conductor, we need only know the order of magnitude of β. β is of the order of magnitude of $\left(\frac{p}{\lambda V^2}\right)^{\frac{1}{2}}$ where λ is the conductivity of the conductor in electromagnetic units and V is the velocity of light.[4] Now for the ordinary metals λ is of the order 10^{-4} while V^2 is of the order of 10^{21}. Hence β is of the order of $\left(\frac{p}{10^{17}}\right)^{\frac{1}{2}}$.

[4] J. J. Thompson, "Recent Researches."

We have also

$$a^2 = 4\pi\lambda\mu ip - R\mu p^2$$

$$a^2 = p\left\{4\pi i\lambda\mu - \frac{p}{V^2}\right\}.$$

$$= p\left\{4\pi i\lambda\mu - \frac{p}{10^{21}}\right\}.$$

Therefore for all ordinary metals, unless the frequency runs into the millions of cycles per second, the second term is negligible compared to the first; hence we are justified in writing, when the conductor is a metal,

$$a^2 = 4\pi\lambda\mu ip.$$

But $m^2 = a^2 + \beta^2$. Since β is of the order of magnitude $\left(\frac{p}{10^{17}}\right)^{\frac{1}{2}}\beta^2$ is negligible compared to α^2, and we may write

$$m^2 = a^2$$

$$im = i\sqrt{4\pi\lambda\mu ip}.$$

Now if we confine our attention to the phenomena occurring at the point $z = 0$ in the conductor, as we may do without loss of generality, we have, setting $z = 0$ in (16) and (18):

$$E_z = (A_1 + A_2)\, J_0(imr)\, \epsilon^{ipt}$$

$$E_R = -\frac{\beta}{m}(A_1 - A_2)\, J_1(imr)\, \epsilon^{ipt} \qquad \text{at } z = 0.$$

The corresponding current densities, denoted by u_z and u_R are:

$$u_z = \lambda (A_1 + A_2)\, J_0(imr)\, \epsilon^{ipt} \tag{19}$$

$$u_R = \lambda \frac{\beta}{m}(A_1 - A_2)\, J_1(imr)\, \epsilon^{ipt} \tag{20}$$

Now for ordinary metals we have shown that $\frac{\beta}{m}$ is of the order of 10^{-8}. Hence, unless the reflected wave is nearly equal to the transmitted wave, in which case $A_1 + A_2$ is a small quantity, u_R, the radial or changing current in the conductor, is very small compared to u_z, the current along the conductor, since u_R contains the factor $\frac{\beta}{m}$. Therefore when large currents are transmitted along the conductor, which is the only case for which the current distribution becomes of practical importance, the radial current density u_R is entirely negligible compared to u_z, the current transmitted along the conductor. We shall, therefore, in our subsequent analysis neglect the radial current and the heating

produced by it. The error introduced by our ignoring u_x is so small as to be of theoretical interest only.

Confining our attention to the current density u_z and dropping subscript, we have

$$u = \lambda A J_0(mr) \epsilon^{ipt} \quad \ldots\ldots\ldots\

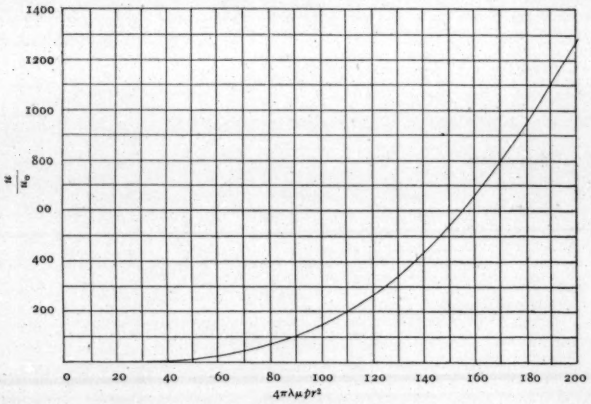
If we write

$$u = u_0 \left\{ a \cos pt + b \sin pt \right\}$$

where u_0 is the current density at the axis and a and b are functions of γr^2, as given by equation (21), the absolute value of the current density is given by

$$| u | = u_0 \sqrt{a^2 + b^2} \quad \ldots\ldots (21)a$$

FIG. 3.



Curve showing $\frac{\text{current density at circumference}}{\text{current density at axis}}$ for cylindrical conductor.

If θ is the angle of lag of the current density at any distance r from axis, behind current density at axis, then

$$\tan \theta = \frac{b}{a} \quad \ldots\ldots (21)b$$

In Fig. 3, $\frac{u}{u_0}$ and in Fig. 4, θ are plotted against γr^2 or $4\pi\lambda\mu pr^2$.

So far we have considered the current density u. The total current transmitted is obviously given by

I = total current transmitted

$$= \int_0^a 2\pi urdr \text{ when } a \text{ is the radius of the conductor.}$$

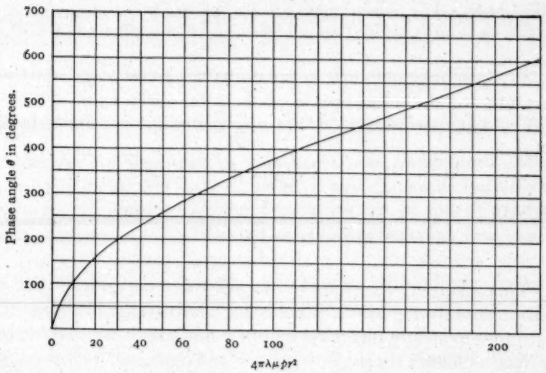
Substituting in our integral the expression for u given by equation (21), we get

$$I = 2\pi\lambda A \left\{ \begin{array}{l} \cos pt \left[\frac{a^2}{2} - \frac{\gamma^2 a^6}{6(2\cdot 4)^2} + \frac{\gamma^4 a^{10}}{10(2\cdot 4\cdot 6\cdot 8)^2} - \cdots\cdots \right. \\ -\sin pt \left[\frac{\gamma a^4}{4(2)^2} - \frac{\gamma^3 a^8}{8(2\cdot 4\cdot 6)^2} + \frac{\gamma^5 a^{12}}{12(2\cdot 4\cdot 6\cdot 8\cdot 10)^2} - \cdots\cdots \right. \end{array} \right.$$

The mean square current, I_m^2, is given by

$$I_m^2 = \frac{(2\pi\lambda A)^2}{2} \left\{ \begin{array}{l} \left[\frac{a^2}{2} - \frac{\gamma^2 a^6}{6(2\cdot 4)^2} + \frac{\gamma^4 a^{10}}{10(2\cdot 4\cdot 6\cdot 8)^2} - \cdots\cdots \right]^2 \\ + \left[\frac{\gamma a^4}{4(2)^2} - \frac{\gamma^3 a^8}{8(2\cdot 4\cdot 6)^2} + \frac{\gamma^5 a^{12}}{12(2\cdot 4\cdot 6\cdot 8\cdot 10)^2} + \cdots\cdots \right]^2 \end{array} \right.$$

FIG. 4.



Curve showing change of phase of current in cylindrical conductor with change of distance from axis.

At first glance this is an extremely difficult expression to handle, but I have found that it can be expressed in the remarkably simple form:

$$\frac{I_m^2}{} = \frac{(2\pi\lambda A)^2}{8} a^4 \left\{ 1 + \sum_{n=1}^{n=\infty} \gamma^{2n} a^{4n} \left(\frac{1}{n+1}\right)\left(\frac{1}{2n+1}\right)\left(\frac{1}{2\cdot 4\cdots 4n}\right)\left(\frac{1}{2\cdot 4\cdots 2n}\right)^2 \right.$$

We now have to consider the heating effect of the current in

the conductor. If u_m^2 is the mean square of the current density, the heating per unit length, W, is given by

$$W = \frac{1}{\lambda} \int_0^a 2\pi (u_m)^2 r dr$$

From equation (21), u_m^2 is equal to

$$u_m^2 = \left(\frac{\lambda A}{2}\right)^2 \left\{ \left[1 - \frac{(\gamma r^2)^2}{(2\cdot 4)^2} + \frac{(\gamma r^2)^4}{(2\cdot 4\cdot 6\cdot 8)^2} - \frac{(\gamma r^2)^6}{(2\cdot 4\cdots 12)^2} + \cdots\cdots \right]^2 + \left[\frac{\gamma r^2}{2^2} - \frac{(\gamma r^2)^3}{(2\cdot 4\cdot 6)^2} + \frac{(\gamma r^2)^5}{(2\cdot 4\cdots 10)^2} - \cdots\cdots \right]^2 \right.$$

This series also admits of a simple formulation. Without going into the detailed work, W, the Joulean evolution of heat per second, may be written

$$W = \frac{\pi\lambda A^2}{2} a^2 \left\{ 1 + \sum_{n=1}^{n=\infty} \left(\gamma a^2\right)^{2n} \left(\frac{1}{2n+1}\right) \left(\frac{1}{2\cdot 4\cdots 4n}\right) \left(\frac{1}{2\cdot 4\cdots 2n}\right)^2 \right. \tag{23}$$

We have now obtained in equations (22) and (23) analytical formulæ for the mean square of the total current and the mean heat product per second. If we now define the resistance by the relation

$$I_m^2 R = W$$

we have

$$R = \frac{W}{I_m^2}$$

This quantity, R, which is the resistance per unit length, we shall denote by R_{ac}, since it holds for alternating currents. The resistance to steady direct current per unit length we shall denote by R_{dc}. Obviously,

$$R_{dc} = \frac{1}{\pi\lambda a^2}$$

As a result of the foregoing definition of R_{ac}, we have, by aid of equations (22) and (23),

$$R_{ac} = R_{dc} \left[\frac{1 + \sum_{n=1}^{n=\infty} \left(\gamma a^2\right)^{2n} \left(\frac{1}{2n+1}\right) \left(\frac{1}{2\cdot 4\cdots 4n}\right) \left(\frac{1}{2\cdot 4\cdots 2n}\right)^2}{1 + \sum_{n=1}^{n=\infty} \left(\gamma a^2\right)^{2n} \left(\frac{1}{2n+1}\right) \left(\frac{1}{2\cdot 4\cdots 4n}\right) \left(\frac{1}{2\cdot 4\cdots 2n}\right)^2 \left(\frac{1}{n+1}\right)} \right] \tag{24}$$

Both numerator and denominator of the foregoing expression are highly convergent series in (γa^2). It is interesting to note that the n^{th} term of the denominator is equal to the corresponding term of the numerator, multiplied by $\frac{1}{n+1}$. When $p = 0$, $\lambda = 0$, and the expression reduces to $R_{ac} = R_{dc}$, as we should expect.

The expression for R_{ac} can be thrown into the form

$$R_{ac} = R_{dc}\left[1 + c_1(\gamma a^2)^2 + c_2(\gamma a^2)^4 + c_3(\gamma a^2)^6 r \cdots\right]$$

and the coefficient—c_1, c_2, c_3—calculated.

This gives

$$\frac{R_{ac}}{R_{dc}} = \left[1 + \frac{1}{12 \cdot 16}(\gamma a^2)^2 - \frac{1}{180 \cdot 16^2}(\gamma a^2)^4 + \cdots\cdots\right]$$

This series is, however, very difficult to calculate for higher terms. I have therefore found it expedient, in calculating the values for the curves, Figs. 5 and 6, to compute the numerator and denominator of equation (24) separately for different values of λa^2. It is interesting to note that $\frac{R_{ac}}{R_{dc}}$, and all the effects due to current distribution are functions of $4\pi\lambda\mu . pa^2$. This renders all curves universally applicable if the dimensions and electrical constants of the conductor and the frequency are given. p, of course, is equal to $2\pi\omega$ where ω is the number of cycles per second.

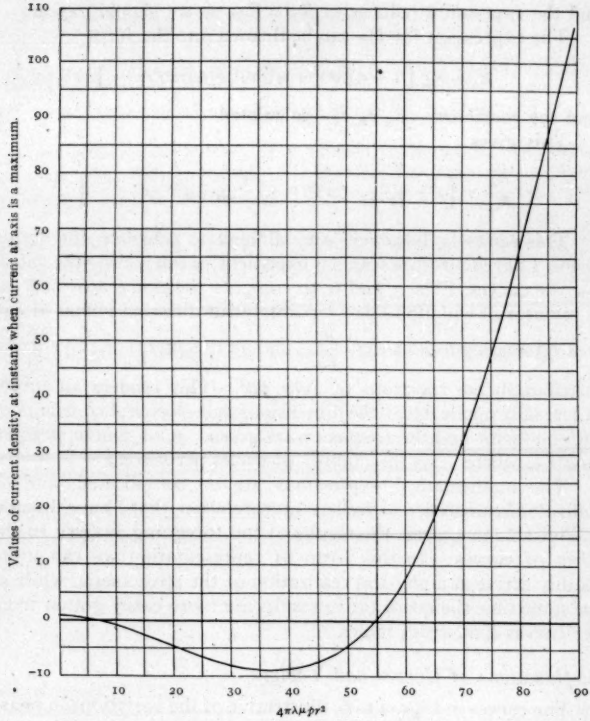
The mathematical expressions are so complicated, and so difficult of calculation as well as interpretation, that I have thought it advisable to express the results of the foregoing analysis in the form of curves. In this form of representation we can more readily arrive at a physical realization of the phenomena, while at the same time the quantitative results are more easily gotten from the curves than from tables.

Explanations of Curves and Tables.

The curves in Figs. 3 to 6, illustrative of the variations in phase and density of the current as we go out from the axis of the conductor, are plotted with values of $4\pi\lambda\mu\, pr^2$ as abscissæ. Curves in Figs. 7 and 8, which give the values of the ratio $\frac{R_{ac}}{R_{dc}}$, are plotted with values of $4\pi\lambda\mu\, pa^2$ as abscissæ, where a is the radius of the

conductor in centimetres. It must be carefully noted that λ, the conductivity, is expressed in absolute units; in substituting in this expression $4\pi\lambda\mu\, pa^2$, the conductivity in practical units must

FIG. 5.



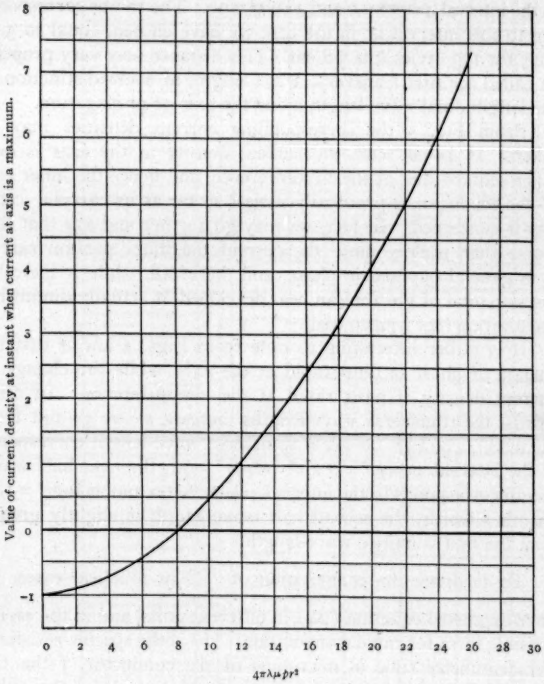
Curve of instantaneous current density in conductor of circular cross-section. $4\pi\lambda\mu\, pa^2$ = one transverse wave-length.

be multiplied by 10^{-9} to convert it to the absolute system of units.

As an illustration of the use of these curves, we shall take a practical example for curves in Figs. 3 and 4. For copper, using the

value of $(5.88)\ 10^5$ as the conductivity in practical units, we have $4\pi\lambda\mu p = \omega\ (4.695)\ 10^{-2}$, where ω is the frequency in cycles per second. Taking the case of 60 ω we have $4\pi\lambda\mu p = 2.82$ approxi-

FIG. 6.



Curve of instantaneous current density in conductor of circular cross-section. $4\pi\lambda\mu pa^2$ = one-half transverse wave-length.

mately. Now by reference to Fig. 4 we see that for a reversal of phase of current density $4\pi\lambda\mu pa^2 = 25.6$. Therefore in the case we have chosen, if $a^2 = \frac{25.5}{2.82}$, $a = 3$ cm., we have the condition

that the current at the axis is in exactly opposite phase from that at the surface. For higher frequencies this condition is attained for corresponding smaller values of conductor radius. Referring again to Fig. 4, we see that for $4\pi\lambda\mu a^2 = 90$, the current at axis and centre are in phase, the phase having changed through 360° in the interval $4\pi\lambda\mu r^2 = 0$ and $4\pi\lambda\mu r^2 = 90$. The radius corresponding to this interval is, in the case we have chosen, equal to 5.65 cm.; for 133 cycles it is 3.8 cm. This distance may very properly be called the first transverse wave-length, in contradistinction to the longitudinal wave-length along the axis of propagation.

From Fig. 3 the corresponding current densities may be taken. If $4\pi\lambda\mu a^2 = 88$, the current density at the axis is only 1 per cent. of that at the circumference, and hence the inner part of the conductor is practically useless so far as its carrying capacity is concerned. In fact, we may go further and say that it is worse than useless, since the current the inner portion carries is in general opposed in phase, and therefore, while it increases the sum total of the Joulean heat developed, it actually diminishes the total current transmitted.

It is rather interesting to note from Figs. 3 and 4 that the change of phase is most rapid at the axis, while the change of current density is most rapid at the circumference. In other words, the transverse wave-lengths increase as we go out from the axis. The first complete transverse wave-length corresponds to the interval $4\pi\lambda\mu pr^2 = 0$ and $4\pi\lambda\mu pr^2 = 90$; the next half wave-length corresponds to the interval $4\pi\lambda\mu pr^2 = 90$ and $4\pi\lambda\mu pr^2 = 185$. In other words, the second half wave-length is slightly greater than the first complete wave-length.

To facilitate the computation of $\frac{R_{ac}}{R_{dc}}$ in practical cases, we have expressed equation (24) in different units, and in the easiest possible form for rapid computation. If ρ is the specific resistance per centimetre cube in microhms of the conductor, f the frequency in cycles per second, μ the permeability, and a the radius of the conductor in centimetres, equation (24) may be written

$$\frac{R_{ac}}{R_{dc}} = \frac{1 + c_1\left(\frac{\mu f a^2}{\rho}\right)^2 + c_2\left(\frac{\mu f a^2}{\rho}\right)^4 + c_3\left(\frac{\mu f a^2}{\rho}\right)^6 + \cdots}{1 + \tfrac{1}{3}c_1\left(\frac{\mu f a^2}{\rho}\right)^2 + \tfrac{1}{3}c_2\left(\frac{\mu f a^2}{\rho}\right)^4 + \tfrac{1}{3}c_3\left(\frac{\mu f a^2}{\rho}\right)^6 + \cdots} \qquad (A)$$

The interesting relation between the coefficients of corresponding terms of numerator and denominator makes the computation of the denominator extremely easy, once the numerator is evaluated. Below is a table of the coefficients $c_1 c_2 \ldots c_6$ to six places:

$$c_1 = \ldots\ldots\ldots\ldots\ldots (6 \cdot 49\ ) \times 10^{-5}$$
$$c_2 = \ldots\ldots\ldots\ldots\ldots (3 \cdot 55\ ) \times 10^{-10}$$
$$c_3 = \ldots\ldots\ldots\ldots\ldots (3 \cdot 24\ ) \times 10^{-16}$$
$$c_4 = \ldots\ldots\ldots\ldots\ldots (1 \cdot 132) \times 10^{-22}$$
$$c_5 = \ldots\ldots\ldots\ldots\ldots (1 \cdot 55\ ) \times 10^{-29}$$
$$c_6 = \ldots\ldots\ldots\ldots\ldots (1 \cdot 073) \times 10^{-36}$$

The coefficients to six places will give the value of $\frac{R_{ac}}{R_{dc}}$ accurately for values of $\left(\frac{\mu f a^2}{\rho}\right)$ not exceeding $(1.3) \times 10^3$. The numbers of terms to which the expansion must be carried depends, of course, on the value of $\left(\frac{\mu f a^2}{\rho}\right)$ for the case chosen.

For values of $\left(\frac{\mu f a^2}{\rho}\right)$ between 1.25×10^3 and 2.5×10^3 the following approximate formula may be used with a high degree of accuracy:

$$\frac{R_{ac}}{R_{dc}} = 2.35 + (1.145)\ (10^{-3}) \left(\frac{\mu f a^2}{\rho}\right) \qquad (B)$$

For values of $\left(\frac{\mu f a^2}{\rho}\right)$ lying between 2.5×10^3 and 3.8×10^3, the approximate formula should be used:

$$\frac{R_{ac}}{R_{dc}} = 2.95 + (.908)\ (10^{-3}) \left(\frac{\mu f a^2}{\rho}\right) \qquad (C)$$

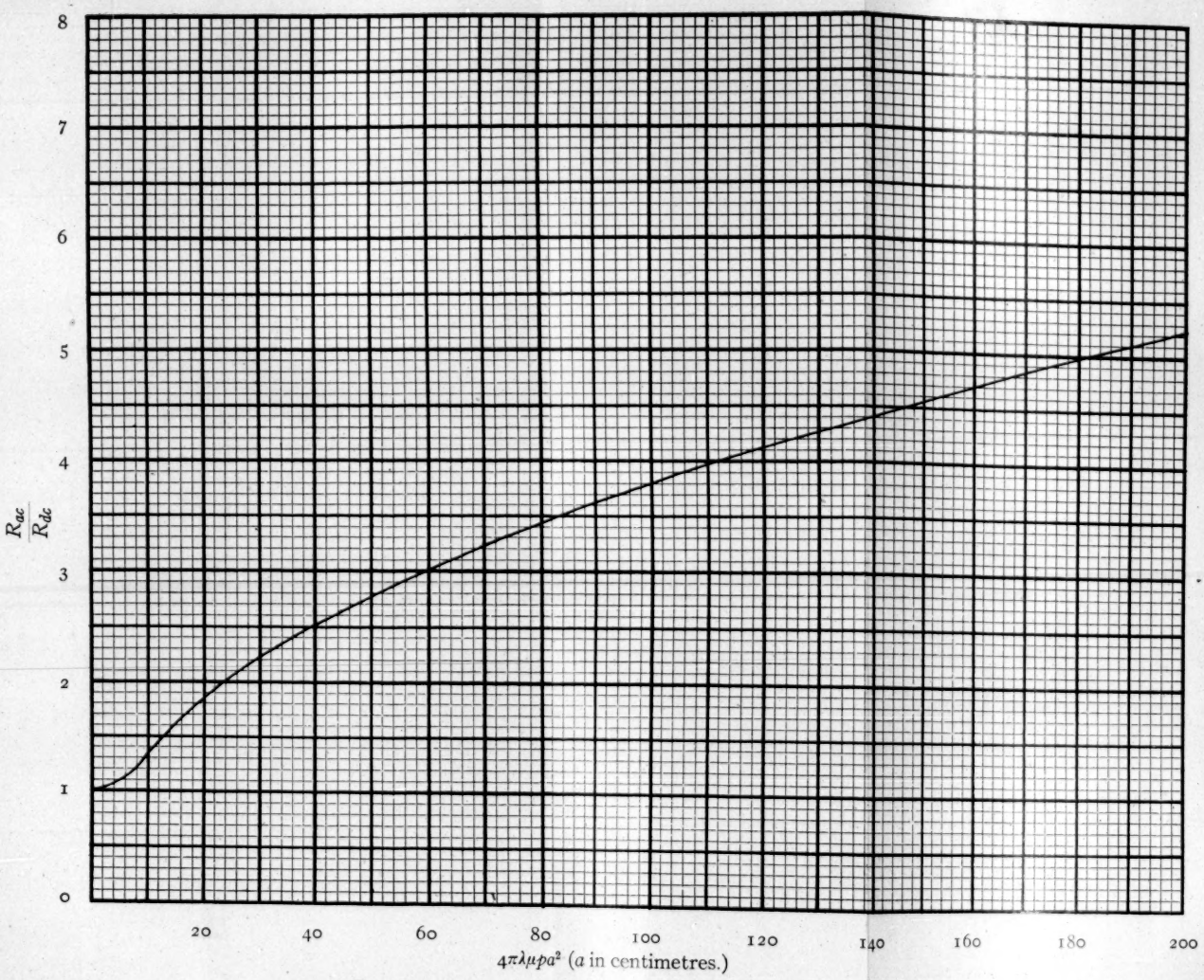
As an example of the use of these formulæ, suppose we wish to determine $\frac{R_{ac}}{R_{dc}}$ for an iron conductor of permeability 1000, specific resistance of 10.4 microhms per centimetre cube, radius 1 centimetre, and frequency 25 cycles. Then we have

$$\frac{\mu f a^2}{\rho} = \frac{(1000)\ (25)\ (1)^2}{10.4} = (2.4)\ (10)^3.$$

We see from the value of $\frac{ufa^2}{\rho}$ that formula B may properly be applied, and by substitution therein we get

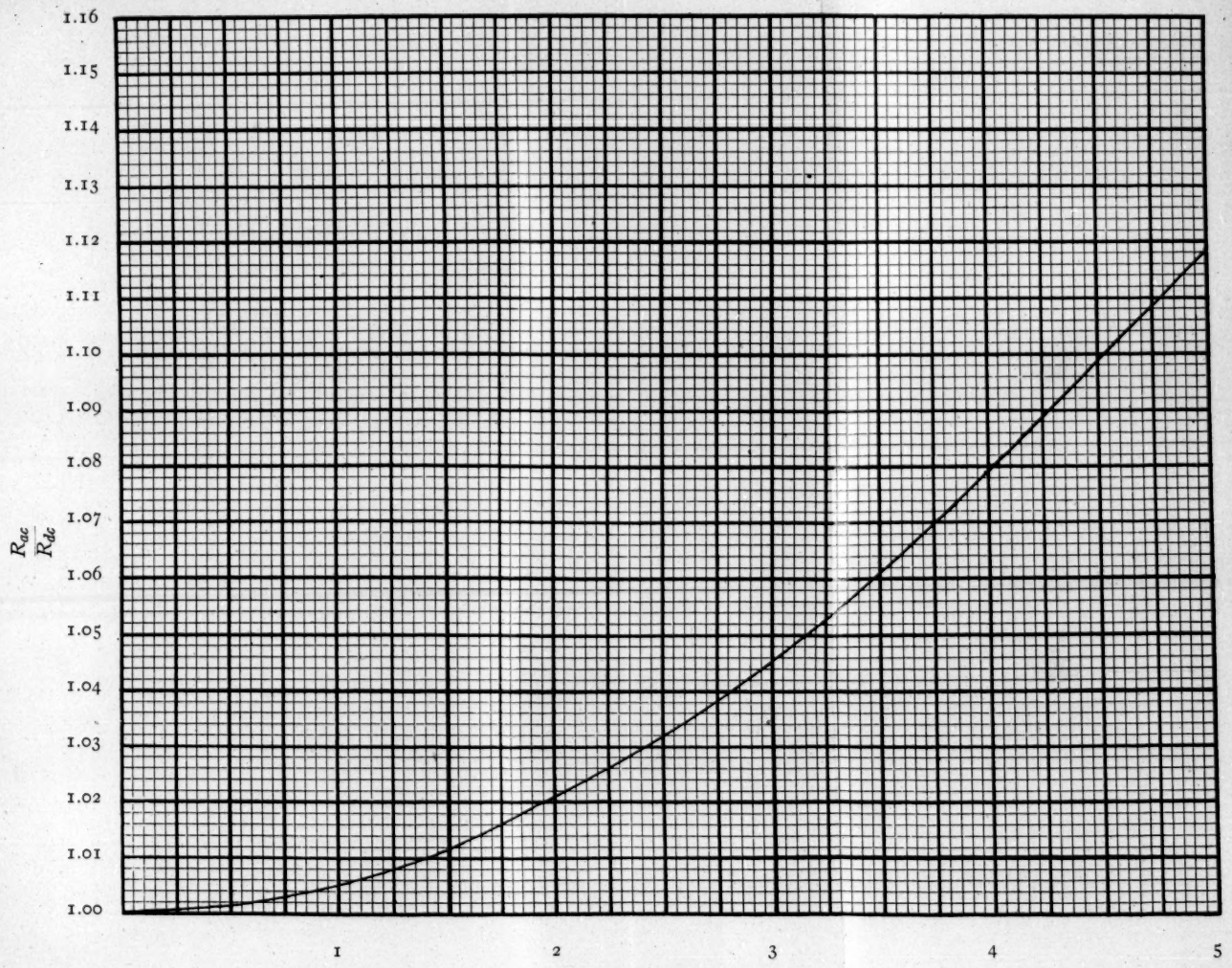
$$\frac{R_{ac}}{R_{dc}} = 1.25 + 2.75 = 4.0.$$

FIG. 7.



Curve showing $\frac{R_{ac}}{R_{dc}}$ as function of $4\pi\lambda\mu pa^2$ for a solid conductor of circular cross-section.

FIG. 8.



Curve showing $\frac{R_{ac}}{R_{dc}}$ as function of $4\pi\lambda\mu pa^2$ for a solid conductor of circular cross-section.

Again, if we have a copper conductor of radius 1 centimetre, specific resistance of 1.7 microhms per centimetre cube, at 60 cycles, $\frac{\mu f a^2}{\rho} = 35.3$. This value of $\frac{\mu f a^2}{\rho}$ is too small to permit of the use of either formula B or C, and we are therefore obliged to substitute in formula A.

Performing the indicated operations, we have

$$c_1\left(\frac{\mu f a^2}{\rho}\right)^2 = .0808$$

$$c_2\left(\frac{\mu f a^2}{\rho}\right)^4 = .00055$$

Subsequent terms become negligibly small, and we have

$$\frac{R_{ac}}{R_{dc}} = \frac{1 + .0808 + .00055}{1 + \frac{1}{2}(.0808) + \frac{1}{3}(.00055)}$$

$$= \frac{1.0813}{1.0407} = 1.04$$

Hence in this case the resistance of the conductor to alternating current is 104 per cent. of its resistance to direct current.

If we choose a conductor of larger radius, operating at a higher frequency, higher terms in formula A become of importance, and the expansion must be carried out further.

In considering these theoretical results it behooves us to scrutinize the assumptions on which they rest, and to determine, in so far as we may, just when they may be safely applied in practice, and when, also, the assumptions depart so far from actual conditions as to vitiate our results. In the first place, we made the assumption that the cylindrical conductor was surrounded by a concentric cylindrical shell of conductive material, the two conductors being separated by the dielectric. This condition is not realized in practice, where we in general have two parallel conductors which are not concentric. Our analysis will not hold theoretically for this case. The complete solution for the case of parallel conductors has never been given, but it is known that higher orders of Bessel's Functions appear in this solution. However, if the radii of the conductors are small compared to the distance between their axes, the departure from the results predicted by the foregoing analysis would be very small, and the skin effect formulæ may be relied upon as very closely accurate.

A final word concerning a point which Dr. Northrup has

brought out very fully; namely, the effect of the variation of permeability and the impossibility of analytically taking this effect into account. Going back to equation (6) in the first part of this section, we should write $\frac{d}{dt}(\mu H)$ instead of $\mu \frac{dH}{dt}$, if μ is variable. Since μ is a function of H, the magnetic force, we have

$$\frac{d}{dt}\left(\mu H\right) = \left(\mu + \frac{\delta_\mu}{\delta H}\right)\frac{dH}{dt}$$

Here, however, analysis must stop. Not only is it impossible to express μ as a single function of H, but also μ is a double-valued function, which renders it impossible to integrate the resulting equation, supposing we could express μ as a function of H.

It follows, therefore, as Dr. Northrup has pointed out, that for ferrous conductors the theoretical formulæ do not accurately hold. They do, however, give the order of magnitude of the effect.

A final limitation to the generality of our formulæ must be noted. This is that the formulæ hold rigorously for long conductors only; in other words, for points in the conductor at a considerable distance from the disturbing effects of terminal conditions. The analytical introduction of this limitation is involved in our assumption that β, the attenuation constant, is very small. Theoretically there are an infinite number of values of β, and these may all be present at the terminal of the conductor. At considerable distances from the terminal, terms containing large values of β are damped out; finally at great distances from the terminal, only the term containing the smallest value of β is present.

The foregoing considerations are of greater theoretical than practical importance; the corrections which theory points out are probably too small to be detected experimentally in conductors longer than two metres.

V. EXPERIMENTAL INVESTIGATION OF THE ALTERNATING-CURRENT RESISTANCE OF IRON RODS.

(a) *The Method, Samples, and Apparatus.*

The object set in planning the experimental part of this research was to make a measurement of the alternating-current

resistance of three samples having different diameters, of round bars of iron of high permeability. As the alternating-current resistance was expected to depend both upon the frequency and the total current, the measurements were made using currents varying from 3 ampères to 43 ampères and for six frequencies. Perhaps the most precise and concise definition of the quantity we call alternating-current resistance is: a quantity numerically equal to the heat production, expressed in watts, per unit length of the conductor divided by the square of the total current flowing. Or we may say that the alternating-current resistance per unit length of a conductor is that quantity which, when multiplied by the mean square value of the total current, will give the power in watts which is being dissipated in unit length of the conductor. It is the quantity which should be employed when multiplying a resistance by the current to obtain in the triangle of electromotive forces, that component of the electromotive force which is in phase with the current.

In assigning a value to the alternating-current resistance obtained under given conditions it is convenient to express it as the ratio which it bears to the ordinary ohmic or direct-current resistance, the temperature being chosen the same in both cases.

The arrangements of the apparatus and method were planned to give a precision to within 1 per cent.

The three samples measured were round iron bars, each about 1.7 metres long. Samples No. 1 and No. 3 were not procured from the same place as No. 2, and were probably of a different kind of iron. This conclusion is made the more likely by determinations which were made of their resistivities. Reduced to microhms per centimetre cube at 25° C., the resistivities (within ¼ of 1 per cent.) were:

Sample No. 1—12.35 microhms.
Sample No. 2—14.63 microhms.
Sample No. 3—12.77 microhms.

Unfortunately, time was not available to determine their permeability curves. The iron was purchased for a good quality wrought iron. The general course of the curves and the characteristics which they exhibit would be affected in magnitude rather than in kind by iron having a different permeability, and it is thought that the above omission does not much lessen the value of

the experimental results and not at all the general conclusions to be drawn. The diameters of the rods were:

Sample No. 1 = 0.953 cm. (⅜").
Sample No. 2 = 1.905 cm. (¾").
Sample No. 3 = 3.493 cm. (1⅜").

At four or five points along each rod small holes were drilled and copper wires driven in to serve as potential points. Thus various lengths of rod could be selected for measurement, the choice of length being made to give the best obtainable sensibility and yet be within the range of the resistance of the mercury standard used in the comparison.

The full description and theory of the general method employed has already been given by Edwin F. Northrup in the *Proceedings of the A. I. E. E.*, June, 1912, and in his book "Methods of Measuring Electrical Resistance," Chapter X. Some minor modifications were employed and a specially-constructed mercury standard was used. This consisted of a tube of hard gray fibre mounted vertically and filled with mercury. The length of the mercury column could be varied. The sliding terminal was so arranged with a fixed scale that the length of the mercury column in circuit could be read to fractions of a millimetre. In reality, however, the resistance used as the standard also included a certain amount of copper. The diagram Fig. 11 shows the arrangement employed.

The more important dimensions of this variable standard are:

Diameter of sliding copper rod = 0.955 cm.

Length of sliding copper rod = 128.4 cm.

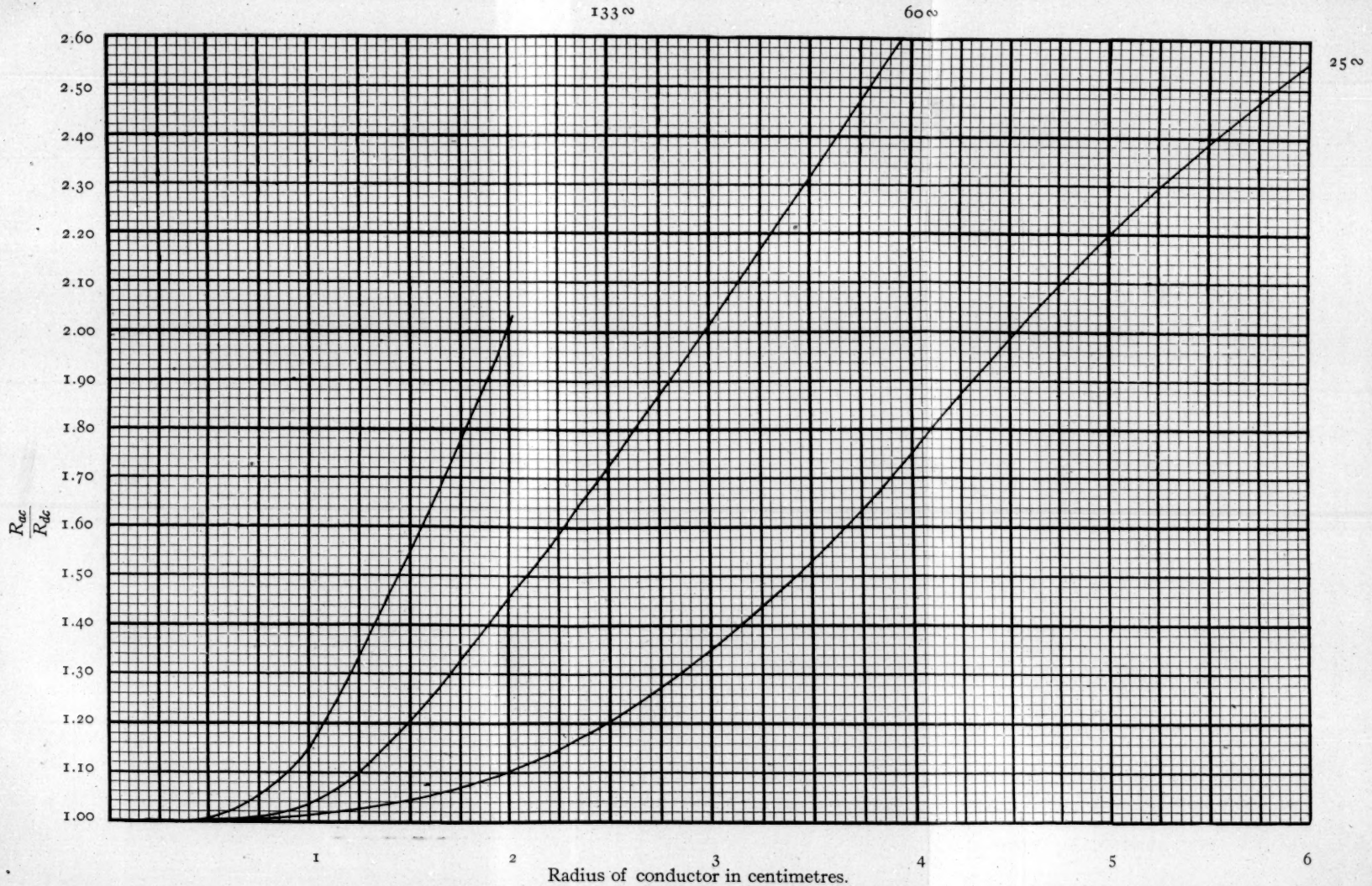
Length of fixed copper rod in bottom of mercury column = 16.4 cm.

Diameter = 0.955 cm.

Inside diameter of fibre tube = 2.00 cm.

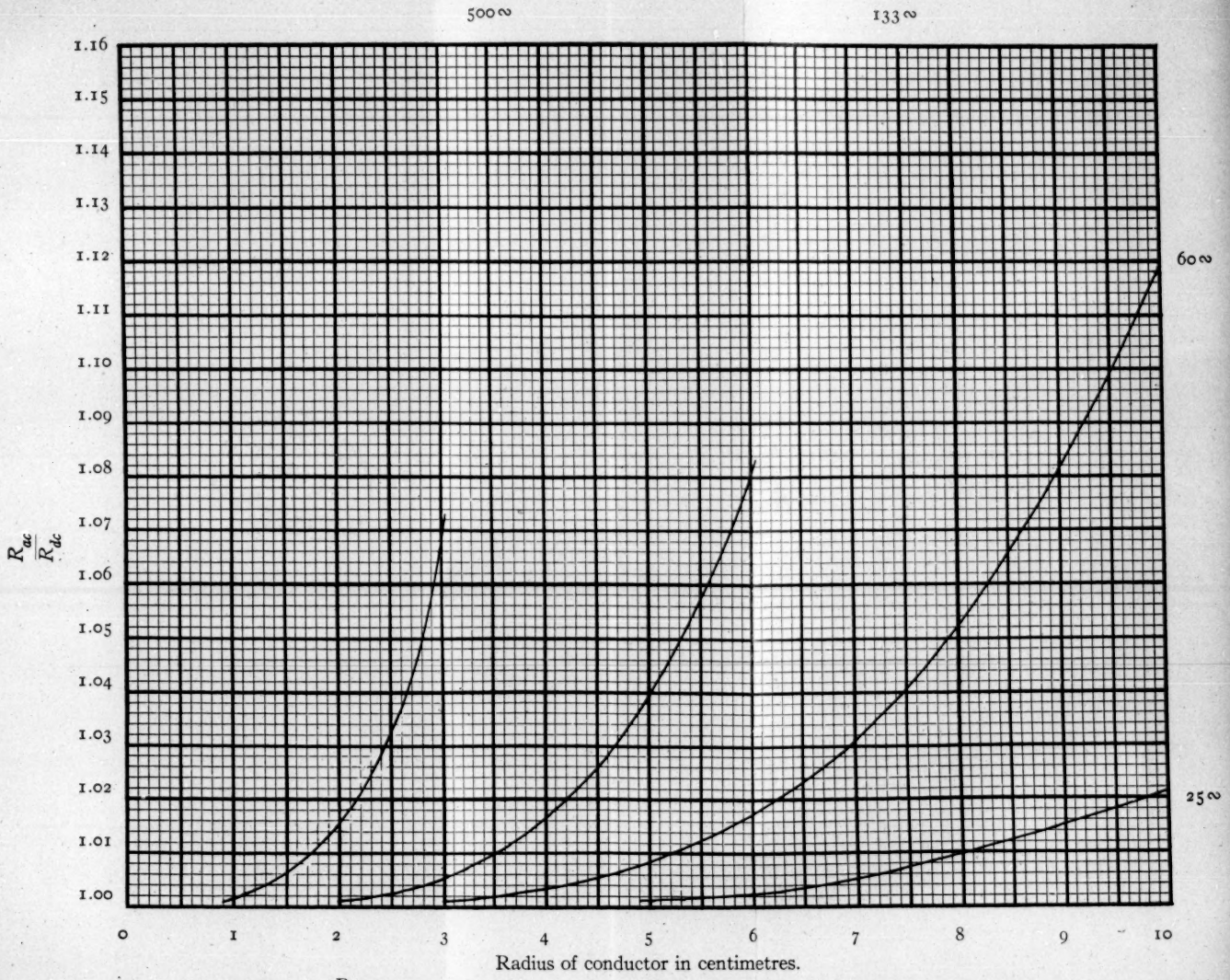
The length of the mercury column, which entered the circuit, could be varied from 0 to 58.5 cm. One end of the iron sample terminated in a short, thick copper plug. This plug was immersed to its full length in one end of a trough of mercury of considerable cross-section, and the lower copper terminal of the mercury column was immersed in the other end of the trough. The distance between the end of the standard and the end of the sample varied from 1.5 to 3 centimetres. One potential point was taken

FIG. 9.



Curve showing $\frac{R_{ac}}{R_{dc}}$ for a copper conductor of circular cross-section. 133, 60, and 25 cycles.

FIG. 10.

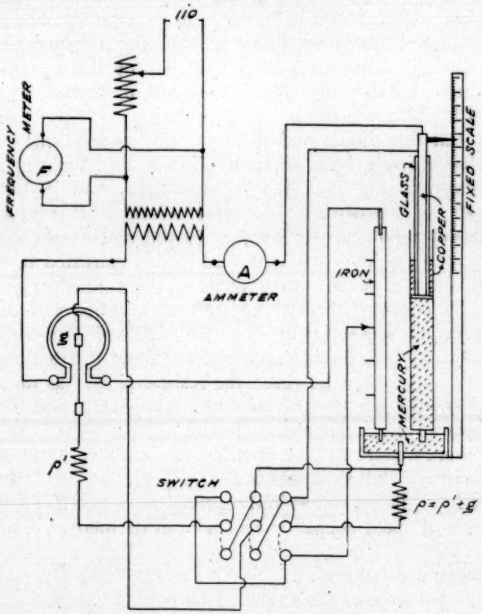


Curve showing $\frac{R_{ac}}{R_{dc}}$ for a mercury conductor of circular cross-section. 500, 133, 60, and 25 cycles.

off at the point where the copper plug inserted in the end of the iron rod entered the mercury trough.

It was assumed, and subsequent analysis proved, that for the frequencies used and the precision sought the skin effect of the standard was entirely negligible,—that is, its alternating-current resistance is sensibly the same as its direct-current resistance. The

FIG. 11.



resistance of the standard for various scale positions was accurately measured with a Kelvin double bridge either immediately before or after making a set of alternating-current resistance measurements, and a curve (in fact, a straight line) was drawn. Then when a balance was obtained, using alternating current, it was only necessary to refer to the curve to obtain the resistance for the setting of the standard which gave the balance and hence the alternating-current resistance for the particular current and frequency used.

The alternating current was supplied from the low-tension side of a transformer with a ratio of 20 to 1. The high-tension side of the transformer was joined to a rotary converter, and this was fed from a storage battery of 120 volts. In this way reasonably steady current and frequency were obtained. To regulate frequency and current, suitable rheostats were placed in the field circuit of the rotary converter and in the high-tension circuit of the transformer.

To measure frequencies between 35 and 65 cycles, a Westinghouse frequency meter was used; and for frequencies outside this range the revolutions per minute were taken, from which the frequency was readily calculated.

The currents were measured with a Westinghouse 5- and 10-ampère scale ammeter in conjunction with a suitable current transformer.

The electro-dynamometer used was a Rowland-deflection type instrument, with an astatic moving system. Its heavy coil would carry from 45 to 50 ampères. It is made and listed by the Leeds and Northrup Company, Catalogue No. 7425. While this instrument served our purpose, its capacity would be too small for a study of the alternating-current resistance of conductors of large cross-section, such as steel rails. An instrument specially fitted to the work of this character could easily be constructed.

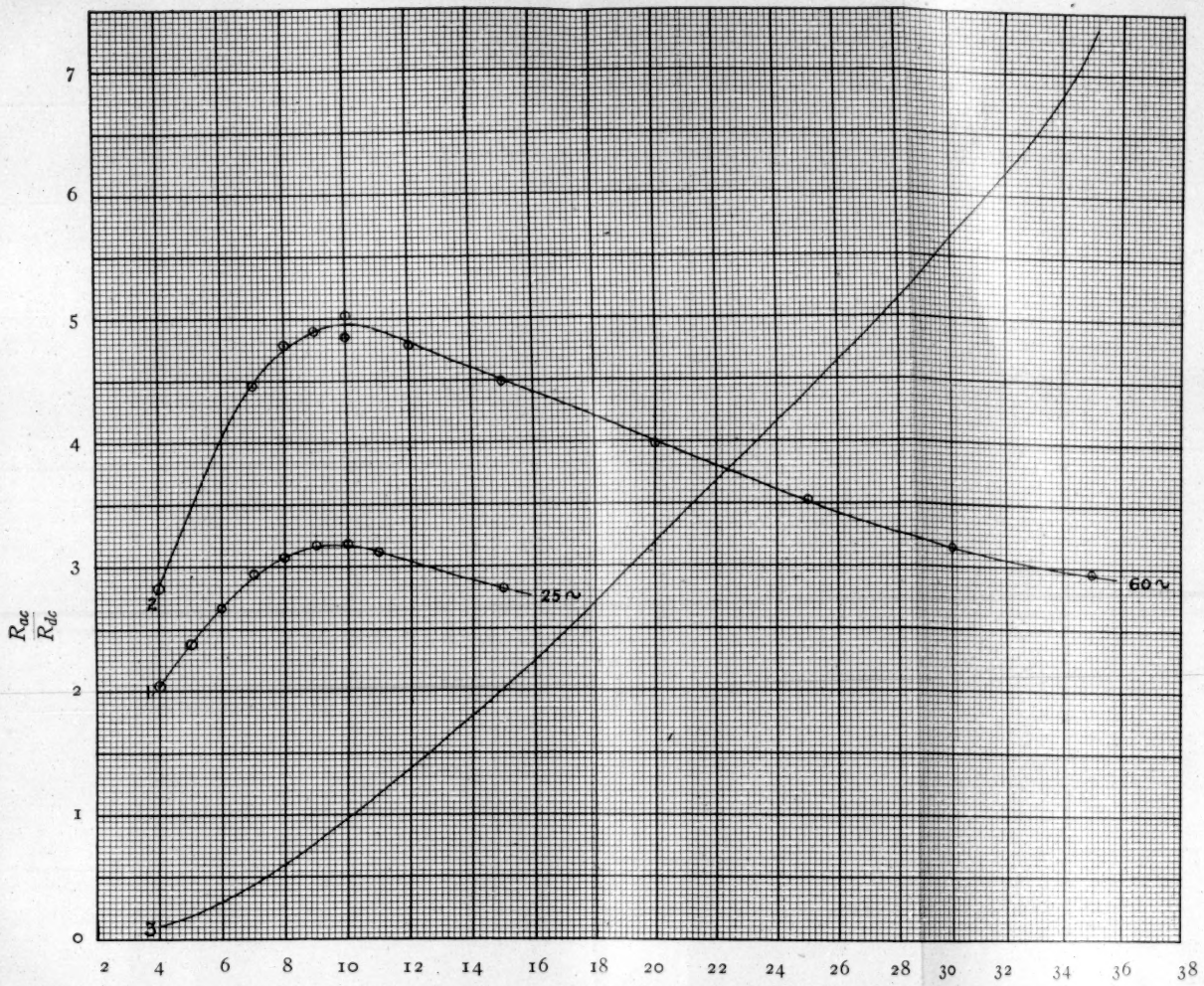
Two or more thermometers were fastened against the surface of the sample being measured by which account was taken of the temperature.

The apparatus as used was connected as shown in the diagram of connections Fig. 11.

As actually set up, all the leads carrying large currents were arranged, as far as possible, non-inductively to eliminate any inductive electromotive forces in the dynamometer potential lead circuits which might lead to erroneous results. With this arrangement a reading was obtained on the dynamometer which was made the same, by varying the mercury standard, whether the swinging coil of the dynamometer was joined across the standard or across the sample. The leads of the swinging coil were then shifted into many positions to determine if there were any induced electromotive forces in this part of the circuits which would affect the reading. None were found.

It was recognized that the non-inductive arrangement of

FIG. 12.



Sample No. 1, ⅜ inch diameter.

standard and sample employed might lead to an error in the measurement of the resistance for any given current, for the current in the standard, being but about 3 centimetres from the iron rod, would cause inductive effects in the iron. As arranged the two were in a vertical position, and it was possible to incline the iron rod to make any angle with the standard. A balance was obtained using the 1⅜-inch bar and 35 ampères and 60 cycles, when both were vertical. The iron rod was then inclined to make an angle of 60° with the standard. An increase in the resistance of a trifle less than 1 per cent. was observed. For lower values of frequency, current, and diameter this error would, we judge, be less. It thus appears that lines of force external to the conductor have but small effect upon the alternating-current resistance, and we think the non-inductive disposition of sample and standard is to be preferred to an inductive disposition because of the reduced chance with the former arrangement of inductive action upon the potential leads. If such were present, considerable error might be caused.

The error which would have resulted from the heating of the rod by the rather large currents used was estimated by taking the temperature of the rod at the time of each reading and making a correction by assuming 0.6 per cent. increase in resistance for each degree Centigrade. The standard did not heat appreciably and also had a lower temperature coefficient, and its resistance was assumed constant.

Using sample No. 2, readings were taken for various currents at frequencies of 75.5, 60, 55, 45, 35, and 25 cycles per second. The frequency of 75.5 cycles was the highest obtainable as the apparatus was arranged. Using samples No. 3 and No. 1, but two frequencies were used, viz., 60 and 25 cycles, these being the more important commercial frequencies. For low currents the results were less accurate on account of the lessened sensibility of the dynamometer with small current. The limitations for high currents were due to the heating of the large coil of the dynamometer and to the voltage available.

Although the direct-current resistance of the samples could have been measured with the dynamometer and circuits as set up, we preferred to measure this resistance in each case by means of a Kelvin double bridge. In the alternating-current resistance determinations the length of iron used was varied, as above stated, but in obtaining the ratio of alternating-current resistance to

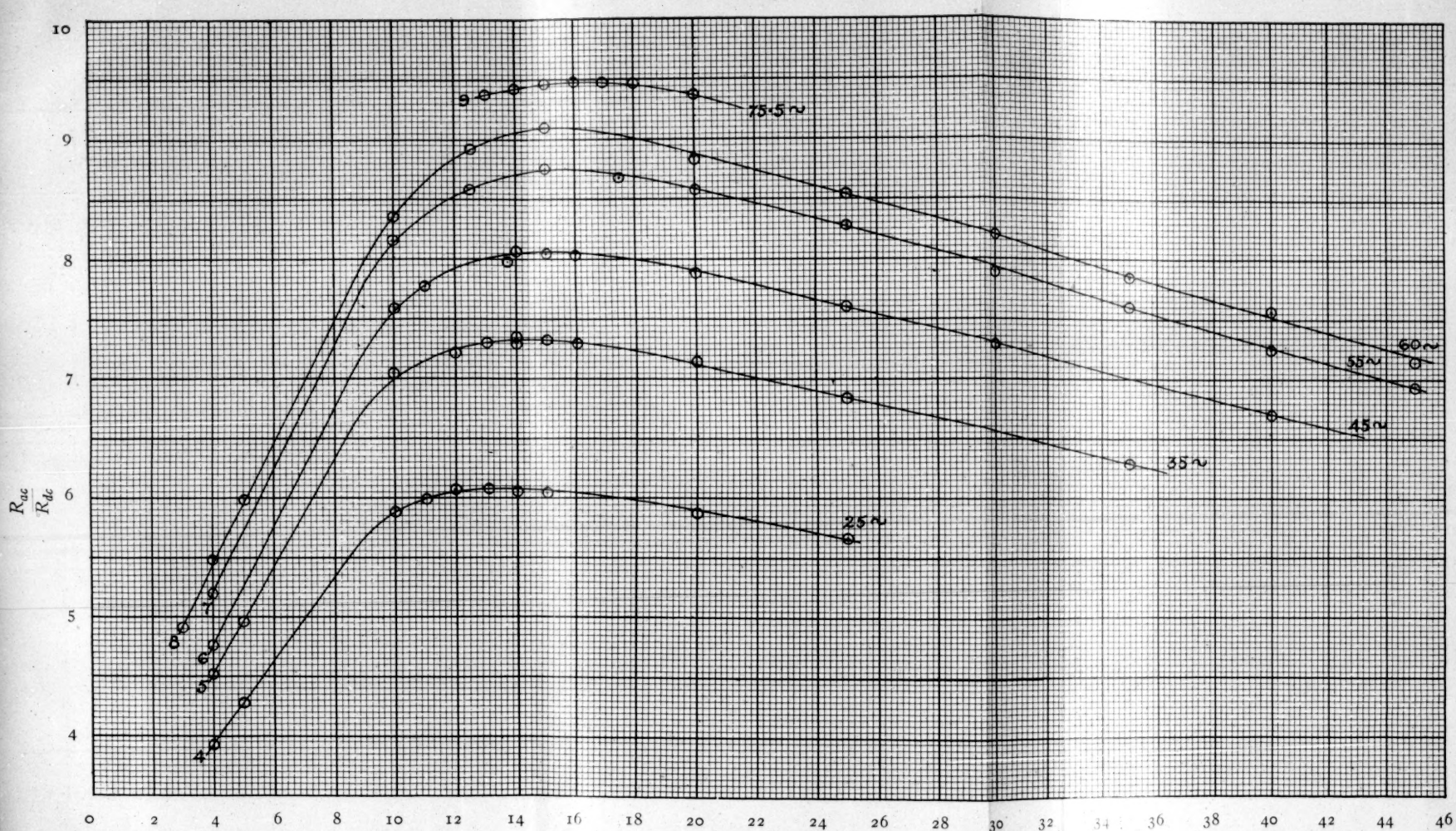
appears from this last that the maximum value of the alternating-current resistance (for 60 cycles at least) is very nearly inversely proportional to the diameter of the rod. Or, stated in another way, the product of the maximum alternating-current resistance and the diameter of the rod is constant. If we express the resistance per centimetre length in ohms and the diameter in centimetres, these products become for samples Nos. 1, 2, and 3, 81.4×10^{-6}, 89.3×10^{-6} and 80.3×10^{-6} respectively. The fact that this product in the case of sample No. 2 is larger than in the other two cases may well be accounted for by observing that its direct-current resistivity is 14.63 microhms, as against 12.35 and 12.77 microhms for samples Nos. 1 and 2.

In the case of conductors of circular cross-section and of non-magnetic material the one general relation which holds exactly is this: the ratio of the alternating-current resistance to the direct-current resistance depends only upon the product of the conductivity of the material, the frequency, and the square of the diameter of the conductor. So long as the product of these three factors remains constant $\frac{R_{ac}}{R_{dc}}$ is unchanged.

VI. THE ENGINEERING FEATURES OF THE EXPERIMENTAL RESULTS.

There are at least three important cases in engineering practice where the skin effect may play a prominent part: in third-rail transmission, in power transmission by means of copper-clad iron conductors, and in the short and very heavy copper conductors which carry large alternating currents from the low-tension side of transformers to electric furnaces. In the last case only, would it be possible to predict without experiment the approximate magnitude of the effect. Furthermore, if conductors of non-magnetic material have a cross-section other than circular, the mathematical calculation of the skin effect would be practically impossible. In all these cases it is very desirable to accumulate experimental data. It is not thought that it would be necessary to measure the ratio, $\frac{R_{ac}}{R_{dc}}$, for every size and shape of iron conductor. It is probable that by measuring a few samples of widely different cross-sections but of similar contour and at different frequencies and with different current values, etc., empirical relations would be observed which would permit quite accurate interpolations to be made for other cross-sections, frequencies, currents, permeabilities, etc.

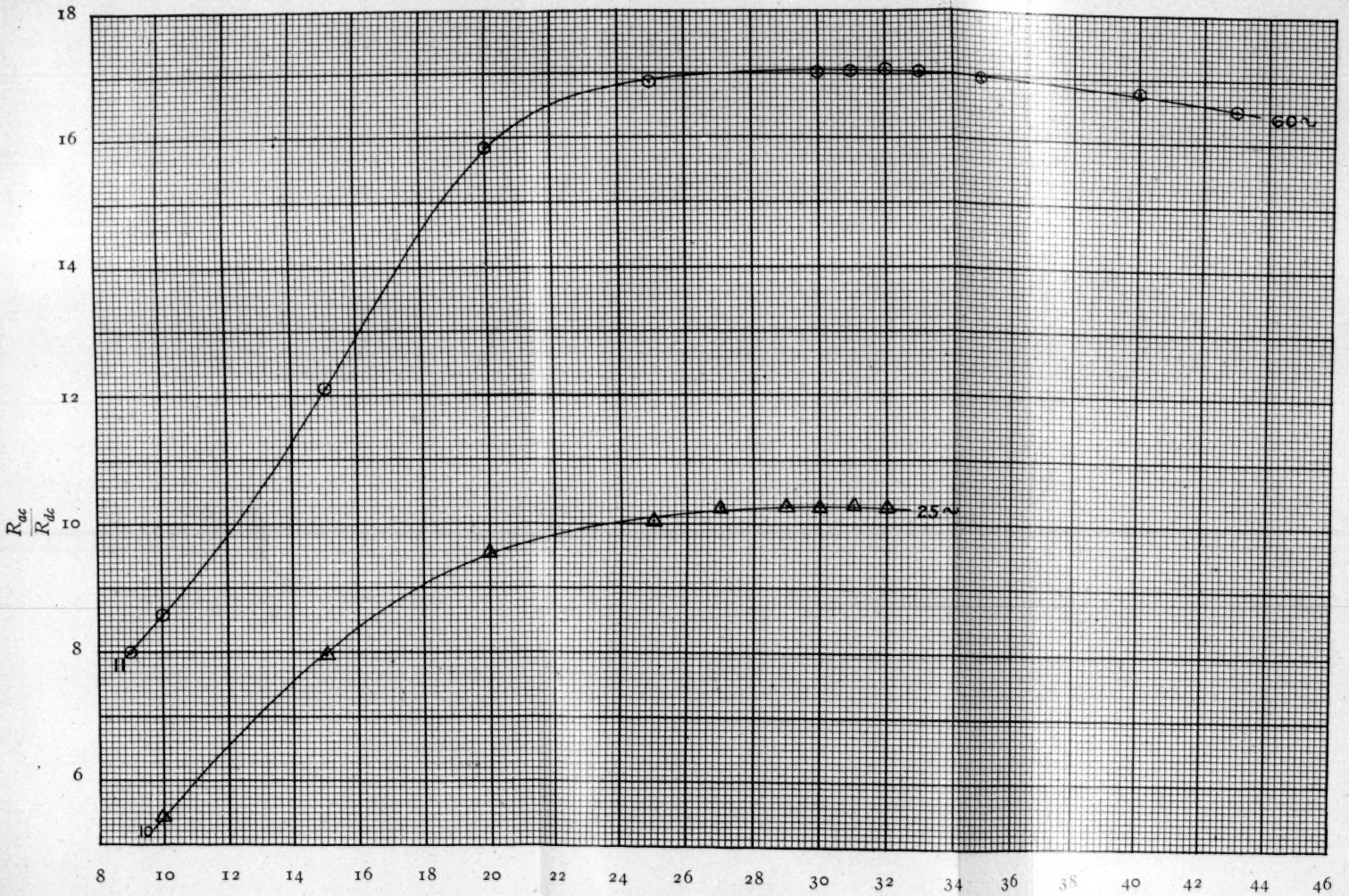
FIG. 13.



Current in amperes.

Sample No. 2, ¾ inch diameter.

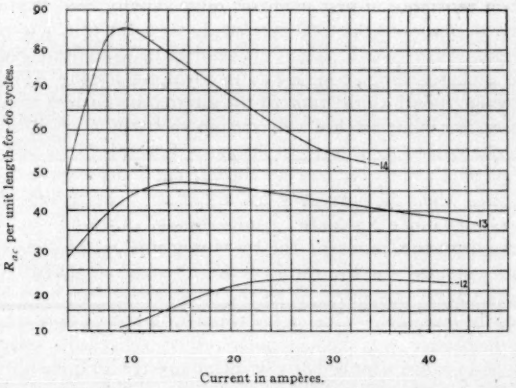
FIG. 14.



Sample No. 3, 1⅜ inches diameter.

least to its greatest value. We note from these curves that, if the iron conductors are made to carry 35 ampères each, at 60 cycles, the watts consumed per centimetre length are: for sample No. 1, $51.5 \times 10^{-6} \times 35^{-2} = 0.063$; for sample No. 2, $40.05 \times 10^{-6} \times 35^{-2} = 0.049$; and for sample No. 3, $22.5 \times 10^{-6} \times 35^{-2} = 0.0276$. The ratio of the last to the first is $\frac{22.5}{51.5} = 0.436$. But the ratio of the cross-section of sample No. 3 to the cross-section of sample No. 1 is 13.44. Thus, if it is required that a

FIG. 15.



conductor transmit 35 ampères at 60 cycles, to reduce the power loss to 0.436, or by a little more than half, we must increase the cross-section,—that is, the weight and cost of the iron,—13.44 times. Considerations of this character reveal the importance in engineering practice of experimentally tracing the curves of alternating-current resistance of conductors of ferrous material which are designed to carry large alternating currents or transmit alternating-current power.

It is interesting to further note that the maximum value of the alternating-current resistance at 60 cycles per unit length for samples Nos. 3, 2, and 1 are 23, 47, and 85.5 microhms respectively. These numbers are in the ratios 1 : 2.04 : 3.71, while the diameters of the samples Nos. 1, 2, and 3 are 0.9525 cm., 1.90 cm., and 3.492 cm. respectively, or are in the ratios 1 : 2 : 3.66. It

appears from this last that the maximum value of the alternating-current resistance (for 60 cycles at least) is very nearly inversely proportional to the diameter of the rod. Or, stated in another way, the product of the maximum alternating-current resistance and the diameter of the rod is constant. If we express the resistance per centimetre length in ohms and the diameter in centimetres, these products become for samples Nos. 1, 2, and 3, 81.4×10^{-6}, 89.3×10^{-6} and 80.3×10^{-6} respectively. The fact that this product in the case of sample No. 2 is larger than in the other two cases may well be accounted for by observing that its direct-current resistivity is 14.63 microhms, as against 12.35 and 12.77 microhms for samples Nos. 1 and 2.

In the case of conductors of circular cross-section and of non-magnetic material the one general relation which holds exactly is this: the ratio of the alternating-current resistance to the direct-current resistance depends only upon the product of the conductivity of the material, the frequency, and the square of the diameter of the conductor. So long as the product of these three factors remains constant $\frac{R_{ac}}{R_{dc}}$ is unchanged.

VI. THE ENGINEERING FEATURES OF THE EXPERIMENTAL RESULTS.

There are at least three important cases in engineering practice where the skin effect may play a prominent part: in third-rail transmission, in power transmission by means of copper-clad iron conductors, and in the short and very heavy copper conductors which carry large alternating currents from the low-tension side of transformers to electric furnaces. In the last case only, would it be possible to predict without experiment the approximate magnitude of the effect. Furthermore, if conductors of non-magnetic material have a cross-section other than circular, the mathematical calculation of the skin effect would be practically impossible. In all these cases it is very desirable to accumulate experimental data. It is not thought that it would be necessary to measure the ratio, $\frac{R_{ac}}{R_{dc}}$, for every size and shape of iron conductor. It is probable that by measuring a few samples of widely different cross-sections but of similar contour and at different frequencies and with different current values, etc., empirical relations would be observed which would permit quite accurate interpolations to be made for other cross-sections, frequencies, currents, permeabilities, etc.

The study of ohmic resistances through moderate ranges of temperature has been pursued quite extensively, but very little has been done in the determination of alternating-current resistances, and it is predicted that this matter will receive the attention it deserves in the near future.

As the skin effect only manifests itself to any appreciable extent in conductors designed to carry power, and as it always acts to increase the loss of power in the conductor, it is an unmitigated evil, and the question naturally arises, how can the skin effect be reduced? In answering this question attention should first be directed to the fact that the skin effect is almost wholly a phenomenon of the lines of induction on the interior of the conductor. It is, therefore, impossible to diminish it by subdividing the conductor into strands which lie close together or by giving to the return conductor any non-inductive disposition. It may, however, be diminished by disposing the material in the form of a tube or a sheet. It can be greatly diminished by decreasing the permeability. Thus steel rails of a certain composition are made which are practically non-magnetic. Even though these show a higher ohmic resistance than ordinary steel rails, they might prove to be very much better carriers of alternating current. If it is possible to divide a conductor into many strands and then intermingle the strands of the return conductor with the strands of the outgoing conductor, the skin effect would be greatly diminished. This procedure might be applied in connection with electric furnaces.

Another question is: Does the ratio of the alternating-current resistance to the direct-current resistance in ferrous materials increase only with the skin effect? It is believed by the writer that this is practically the main cause of the increase, but that the hysteresis of the iron must play a small part in increasing the ratio. The true magnitude of this effect can, however, only be determined by measurement.

The writer believes that in the present paper we have only scratched the surface of this very complex and extensive subject, and he suggests that the experimental investigation of the skin effect and alternating-current resistance would prove an instructive and fruitful line of endeavor for many engineering students who are in search of subjects for theses.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J., Nov. 1, 1913.

PROPORTIONING OF LONG-SPAN TRUSS AND CANTILEVER BRIDGES.*

BY

JOSEPH MAYER,

Assistant Engineer, Quebec Bridge Commission, Montreal, Canada.

The load W_μ produces at the distance z from the end of the cantilever arm a moment

$$M_z = \int_{x=0}^{x=z} \mu (z-x)(AM_x + BS_x)\, dx$$

$$= \mu \int_{x=0}^{x=z} (z-x)\left[A\,\frac{x(l_s+x)}{2} + B\,(x+\tfrac{l_s}{2})\right] dx$$

$$= \mu\left[A\left(\frac{Z^4}{24} + \frac{Z^3 l_s}{12}\right) + B\left(\frac{Z^3}{6} + \frac{Z^2 l_s}{4}\right)\right]$$

Changing Z to x and using a more accurately descriptive index for M_x, we obtain

$$M_x^{d_1 c} = \frac{\mu x^2}{24}\left\{Ax\,(x+2l_s) + 2B\,(2x+3l_s)\right\}$$

The symbol $M_x^{d_1 c}$ indicates the moment at the distance x from the end of the cantilever arm produced by the weight of the cantilever truss required to carry the moving load μ per foot of bridge. The effect of the weight of the suspended span is not included.

The load W_μ produces at the distance x from the end of the cantilever arm a shear

$$S_x^{d_1 c} = \int_0^x \mu\left\{\frac{Ax\,(l_s+x)}{2} + B\,(x+\tfrac{l_s}{2})\right\} dx$$

or

$$S_x^{d_1 c} = \frac{\mu x}{12}\left\{Ax\,(2x+3l_s) + 6B\,(x+l_s)\right\}$$

*Concluded from page 64.

The moving load μ_s per foot of suspended span gives a dead load $\mu_s C_m l_s^2$ and at the end of the cantilever arm a load

$$\mu_s \frac{C_m l_s^2}{2}$$

This load gives at the distance x from the end of the cantilever arm a moment

$$M_x^{d_1 s} = \frac{\mu_s C_m l_s^2 x}{2}$$

Adding this to the moment $M_x^{d_1 c}$, we obtain

$$M_x^{d_1} = \frac{\mu x^2}{24}\left\{ Ax(x + 2l_s) + 2B(2x + 3l_s) + \frac{12\mu_s C l_s^2}{x \cdot \mu} \right\}$$

This is the moment produced at the distance x from the end of the cantilever arm by the weights of the suspended span truss and the cantilever truss required for the moving load and impact stresses.

For the corresponding shear we obtain

$$S_x^{d_1} = \frac{\mu x}{12}\left\{ Ax(2x + 3l_s) + 6B(x + l_s) + \frac{6\mu_s}{x\mu} C_m l_s^2 \right\}$$

The weight of steel in the trusses in the length dx at the distance x from the end of the cantilever arm to carry the first dead loads due to the trusses for the moving load stresses from the loads μ_s and μ_d is

$$dW_\mu^{d_1} = V(AM_x^{d_1} + BS_x^{d_1})\, dx$$

The equivalent uniform slowly moving load $\mu = m_r(1 + I) + m_h$ for the chords is for long spans but little larger than the load per foot behind the engines plus the highway load per foot.

The floor-beam hangers and other members carrying but one panel load or less have a larger impact and larger excess from the engines than the chords. These are allowed for in the coefficient of B. In the trusses required for the dead loads these excesses do not occur. The weight of the trusses to carry a given dead load is, therefore, smaller than that required to carry an equal slowly

moving load μ. To allow for this fact the coefficient V is introduced; it is smaller than 1.

Introducing in the last equation the above values for M_x^d and $S_x^{d_1}$ we obtain

$$dW_\mu^{d_1} = V\Big[\frac{\mu A x^2}{24}\Big\{Ax\,(x+2l_s) + 2B\,(2x+3l_s) + \frac{12\mu_s}{x\cdot\mu}\,C_m l_s^2\Big\} + \frac{\mu Bx}{12}\Big\{Ax\,(2x+3l_s) + 6B\,(x+l_s) + \frac{6\mu_s}{x\mu}\,C_m l_s^2\Big\}\Big]\,dx$$

or

$$dW_\mu^{d_1} = \frac{\mu V}{24}\,dx\Big[A^2x^3\,(x+2l_s) + 4ABx^2\,(2x+3l_s) + 12B^2x(x+l_s) + \frac{12\mu_s}{\mu}\,C_m l_s^2\,(Ax+B)\Big]$$

$$W_\mu^{d_1} = \int_o^{l_c} dW_\mu^{d_1} = \frac{\mu V l_c}{240}\Big\{A^2 l_c^3\,(2l_c+5l_s) + 20AB l_c^2\,(l_c+2l_s) + 20B^2 l_c\,(2l_c+3l_s) + 60\frac{\mu_s}{\mu}\,C_m l_s^2\,(Al_c+2B)\Big\}$$

This is the weight of steel required in the cantilever arm to carry $W\mu$ and the suspended span trusses for the moving load and impact stresses.

If for $\mu_s C_m l_s^2$, which is the weight of steel in the trusses of the suspended span required for the moving load and impact stresses, we place the whole weight W_s of the suspended span, replacing $\mu_s C_m l_s^2$ by W_s, then we obtain

$$dW_\mu^{d_1} = \frac{\mu V dx}{24}\Big\{A^2x^3\,(x+2l_s) + 4ABx^2\,(2x+3l_s) + 12B^2x\,(x+l_s) + \frac{12W_s}{\mu}\,(Ax+B)\Big\}$$

and for $W_\mu^{d_1}$ we obtain

$$W_\mu^{d_1} = \frac{\mu V l_c}{240}\Big[Al_c^2\Big\{Al_c\,(2l_c+5l_s) + 20B\,(l_c+2l_s)\Big\} + 20\,B^2 l_c\,(2l_c+3l_s) + \frac{60W_s}{\mu}(Al_c+2B)\Big] \cdots\cdots\cdots (50)$$

This is the weight of steel required in the cantilever trusses to carry W_μ and W_s.

From $dW_\mu^{d_1}$ we obtain $dW_\mu^{d_2}$ and $W_\mu^{d_2}$ in a similar manner as $dW_\mu^{d_1}$ and $W_\mu^{d_1}$ were obtained from $dW\mu$ and W_s.

And by the same process $W_\mu^{d_3}$ and $W_\mu^{d_3}$ were obtained by repeated integration. They are given in the foregoing text.

LATERALS AND CROSS BRACING.

The formulas for the weights of the laterals and the cross bracing of the suspended span and the weights of the cantilever arm needed to carry them have already been developed. It remains to calculate the weights of the laterals and of the cross bracing of the cantilever arm and the weights of the trusses needed to carry them. The wind pressure at the end of the cantilever arm comes from the suspended span end and is $p_{ws}\frac{l_s}{2}$.

The wind pressure per foot of bridge along the cantilever arm is $p_{wc} = a + bx$, where x is the distance in feet from the end of the cantilever arm.

$p_{ws} = p_{ws}^{t} + p_{ws}^{c} = 900 + 1.4l_s$ for the heaviest loads.

For lighter loads see equations (32) to (35).

The wind shear at the distance x from the end of the cantilever arm is

$$S_w^x = p_{ws}\frac{l_s}{2} + \int_0^x (a + bx)\,dx = p_{ws}\frac{l_s}{2} + \frac{x}{2}(2a + bx)$$

The wind moment at the distance Z from the end of the cantilever arm is

$$M_w^z = p_{ws}\frac{l_s}{2}Z + \int_0^z (Z - x)(a + bx)\,dx = p_{ws}\frac{l_s}{2}Z + \frac{aZ^2}{2} + \frac{bZ^3}{6}, \text{ or,}$$

replacing Z by x, $M_w^x = p_{ws}\frac{l_s}{2}x + \frac{x^2}{6}(3a + bx)$

In very long spans of necessarily large width it is economical to place the railroad tracks far apart. In this case the loading of one track only produces, with top and bottom lateral systems, large stresses in the laterals and the cross bracing. These can be avoided by using only a bottom lateral system and by carrying the wind pressures acting above the bottom laterals to them by cross bracings at the intermediate posts. In this case the whole wind pressure must be carried by the bottom laterals.

The weight of the lateral system for a length dx at the distance x from the end of the cantilever arm is

$$dW_l = dW_{lch} + dW_{ld}$$

The lateral chord stress is

$$\frac{M_w^x}{B_r} = \frac{1}{B_r}\left\{ p_{ws}\frac{l_s}{2}x + \frac{x^2}{6}(3a + bx)\right\}$$

The weight of the length dx of both lateral chords is

$$dW_{lch} = \frac{6.8a_c}{B_r U_c}\left\{ p_{ws}\frac{l_s}{2}x + \frac{x^2}{6}(3a + bx)\right\} dx$$

With the usual double-system stiff diagonal laterals, the weight of the lateral diagonals is

$$dW_{ld} = \frac{6.8a_l}{U_l \sin 2\beta_l}\left\{ p_{ws}\frac{l_s}{2} + \frac{x}{2}(2a + bx)\right\} dx$$

and $$W_l = \int_o^{l_c} dW_{lch} + dW_{ld}$$

$$= \frac{6.8a_c l_c^2}{24 B_r U_c}\left\{ 6p_{ws}l_s + l_c(4a + bl_c)\right\}$$

$$+ \frac{6.8a_l l_c}{6U_l \sin 2\beta_l}\left\{ 3p_{ws}l_s + l_c(3a + bl_c)\right\}$$

The cross bracing weighs about twice as much as the lateral diagonals, or $W_{cb} = 2W_{ld}$, and $W_w = W_l + W_{cb}$.

Therefore

$$W_w = \frac{6.8a_c l_c^2}{24 B_r U_c}\left\{ 6p_{ws}l_s + l_c(4a + bl_c)\right\}$$

$$+ \frac{3.4a_l l_c}{U_l \sin 2\beta_l}\left\{ 3p_{ws}l_s + l_c(3a + bl_c)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (53)$$

This lateral system and cross bracing must be carried by a truss. The formulas for the successive dead loads and the weights of these trusses are derived in a similar manner as those for the moving load trusses. They are given in equations (54) to (58).

The floor of the cantilever arm is practically a uniform load per lineal foot of bridge; it must also be carried by a truss. The shear at the distance x from the end of the cantilever arm is $S_f^x = fx$. The moment at the distance x from the end of the cantilever arm is $M_f^x = \frac{fx^2}{2}$. The weight of the length dx at the distance x from the end of the truss carrying the floor is

$$dW_f = V\,(AM_x + BS_x)\,dx$$

$$dW_f^{d_1} = \frac{Vfx}{2}\,(Ax + 2B)\,dx$$

$$W_f^{d_1} = \int_{x=o}^{x=l_c} dW_f^{d_1} = \frac{Vfl_c^2}{6}\,(Al_c + 3B) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (59)$$

The weight of this truss is carried by a truss the weight of which is $W_f^{d_2}$ given in equation (60).

CANTILEVER ARMS WITH INCLINED CHORDS.

With straight inclined chords the height at the distance from the end of the cantilever arm is $h_x = a + bx$. The moment from the moving load μ per foot of bridge at the distance x from the end of the cantilever arm is $M_x = \mu\,\frac{x}{2}\,(x + l_3)$

The resulting chord stress is

$$\frac{\mu \cdot x\,(x + l_3)\sec\delta}{2\,(bx + a)}$$

where δ is the angle of the chord with the horizontal.

The tension chord section is

$$\frac{\mu x\,(x + l_3)\sec\delta_t}{2\,(bx + a)\,U_t}$$

The weight of the tension chord is for the span length dx

$$\frac{3.4\mu x\,(x + l_3)\,a_t\sec^2\delta_t}{2\,(bx + a)\,U_t}\,.\,dx$$

The weight of the whole tension chord for the load μ is

$$W_{tch} = \int_o^{l_c} \frac{3.4\mu a_t \sec^2 \delta_t \cdot x\,(x + l_s)}{2U_t\,(bx + a)}\,dx$$

$$\frac{x^2 + l_s x}{bx + a} = \frac{x}{b} + \frac{bl_s - a}{b^2} - \frac{a\,(bl_s - a)}{b^2\,(a + bx)}$$

$$\int_o^{l_c} dx\,\frac{x\,(x + l_s)}{bx + a} = \int_o^{l_c} \left[\frac{x}{b} + \frac{bl_s - a}{b^2} - \frac{a(bl_s - a)}{b^2} \cdot \frac{1}{a + bx}\right] dx$$

$$= \frac{lc^2}{2b} + \frac{(bl_s - a)\,l_c}{b^2} - \frac{a\,(bl_s - a)}{b^3} \cdot l_n \cdot \frac{a + bl_c}{a}$$

$a + bl_c = h_p$ = height at pier,

therefore $b = \dfrac{h_p - a}{l_c}$.

If $g_{t_1} = \dfrac{3.4\mu a_t \sec^2 \delta_t}{2U_t}$ then

$$W_{i\cdot t\cdot ch} = g_{t_1} l_c^3 \left\{ \frac{3h_p - 5a}{2\,(h_p - a)} - \frac{a\,(h_p - 2a)}{(h_p - a)^3}\, l_n \frac{h_p}{a} \right\}$$

If $K_1 = \displaystyle\int_o^{l_c} \frac{x\,(x + l_s)}{bx + a} = l_c^3 \left\{ \frac{3h_p - 5a}{2\,(h_p - a)^2} - \frac{a\,(h_p - 2a)}{(h_p - a)^3}\, l_n \frac{h_p}{a} \right\}$

then $W_{i\cdot t\cdot ch} = g_{t_1} K_1$

$W_{i\cdot c\cdot ch} = g_{c_1} K_1$

where $g_{c_1} = \dfrac{3.4 a_c \mu \sec^2 \delta_c}{2U_c}$

$W_{i\cdot ch} = (g_{c_1} + g_{t_1})\,K_1$ where $W_{i\cdot ch}$ is the weight of the two chords of a cantilever arm required for the moving load μ.

Fully written out the last equation becomes:

$$W_{i\cdot ch\cdot} = l_c^3 \left\{ \frac{3h_p - 5a}{2\,(h_p - a)^2} - \frac{a\,(h_p - 2a)}{(h_p - a)^3}\, l_n \frac{h_p}{a} \right\} \times \frac{3.4\mu}{2} \times \left\{ \frac{a_t \sec^2 \delta_t}{U_t} + \frac{a_c \sec^2 \delta_c}{U_c} \right\}$$

For $l_n \frac{h_p}{a}$ we may also write $2.3026 \log \frac{h_p}{a}$.

The shear at the distance x from the end of the cantilever arm is $S_x = \mu\left(x + \frac{l_s}{2}\right)$. The shear is partly in the chords.

The shear in the two chords is

$$S^x_{ch} = \frac{\mu x\,(x + l_s)}{2\,(bx + a)}\,(\tan \delta_t + \tan \delta_c)$$

$\tan \delta_t + \tan \delta_c = b$, therefore

$$S^x_{ch} = \frac{\mu b x\,(x + l_s)}{2\,(bx + a)}$$

The shear in the web is, therefore,

$$S^x_{web} = S_x - S^x_{ch} = \mu\left\{x + \frac{l_s}{2} - \frac{bx\,(x + l_s)}{2\,(bx + a)}\right\}$$

The shear area of the web is

$$\int_o^{l_c} S^x_{web}\,dx = \mu \int_o^{l_c} \left\{x + \frac{l_s}{2} - b\,\frac{x\,(x + l_s)}{2\,(bx + a)}\right\} dx$$

$$= \frac{\mu}{2}\left\{l_c\,(l_c + l_s) - bK_1\right\} = S^a_{web}$$

The weight of the web is

$$W_{i\,.\,web} = BS^a_{web} + \text{pier post.}$$

The pier post, here assumed to be vertical, carries the shear of the top chord to the main pier.

The shear in the top chord of the cantilever arm at the pier is

$$\frac{\mu l_c\,(l_s + l_c)\tan \delta_t}{2\,h_p}$$

The corresponding weight of the post is

$$\frac{3.4 a_{\delta_2}\,\mu l_c\,(l_s + l_c)\tan \delta_t}{2 U_{c_3}}$$

The weight of the web is, therefore,

$$W_{i\text{-}web} = \frac{\mu}{2} B \left\{ l_c(l_c + l_s) - bK_1 \right\} + \frac{3.4 a_{c_2} \mu l_c (l_c + l_s) \tan \delta_t}{2 U_{c_2}}$$

and for $l_c = l_s$

$$W_{i\text{-}web} = \mu B \left\{ l_c^2 - \frac{b}{2} K_1 \right\} + \frac{3.4 a_{c_2} \mu l_c^2 \tan \delta_t}{U_{c_2}}$$

Where U_{c_2} is the unit stress for compression in the pier post.

To obtain the relative weight of the trusses of cantilever arms with inclined and with parallel chords we first calculate the weights of both kinds of trusses required for the moving load and impact stresses, and after this the weights of the two kinds of trusses required for the dead-load stresses.

The assumed moving loads are equivalent uniform loads per foot of bridge, the assumed dead loads are uniform for the suspended span, but increasing from the end of the cantilever arm toward the pier in a definite manner, approximating the increase in the actual dead loads.

We compare trusses with inclined chords of such heights at the end and the main pier as will give approximately the least weight of steel with trusses having parallel chords, and of such height as will give the same weight of chords as the trusses with inclined chords.

For very long spans the most economical suspended span has approximately the length of the cantilever arm; we therefore take for our comparison $l_s = l_c$.

We also take $a = \frac{l_c}{9}$ $\qquad h_p = 4\frac{1}{2} a = \frac{l_c}{2}$.

This gives $b = \frac{h_p - a}{l_c} = \frac{3.5}{9} = 0.3889$.

We obtain then $K_1 = 2.333 l_c^2$.

If we take $\delta_c = \delta_t = \delta$ then

$$g_{c_1} + g_{l_1} = \frac{\mu \sec^2 \delta}{2} 3.4 \left(\frac{a_c}{U_c} + \frac{a_l}{U_l} \right)$$

$$W_{i\text{-}ch} = k_1 (g_{c_1} + g_{l_1}) = 1.1665 \, l_c^2 \mu \sec^2 \delta \times 3.4 \left(\frac{a_c}{U_c} + \frac{a_l}{U_l} \right)$$

For parallel chords we have

$$W_{p.ch} = \frac{5}{12} l_c^3 \mu A \qquad \text{where } A = \frac{3.4}{h}\left(\frac{a_c}{U_c} + \frac{a_t}{U_t}\right)$$

To find what height h with parallel chords gives the same weight of chords as the heights a and $4\frac{1}{2}a$ at the end of the cantilever arm and at the pier we have

$$1.1665 \sec^2 \delta = \frac{5}{12}\frac{l_c}{h}$$

This gives

$$h = \frac{5 l_c}{12 \times 1.1665 \sec^2 \delta}$$

$$\tan \delta = \frac{1.75}{9} = 0.19447 \quad \sec^2 \delta = 1.038$$

$$\therefore \quad h = \frac{l_c}{2.906} = 0.3441 l_c$$

The weight of the web with inclined chords is

$$W_{i\,web} = \mu B l_c^2 (1 - 0.1944 \times 2.333) + \frac{3.4\,\mu l_c^2\, 1.5 \times 0.1944}{19600}$$

$$= \mu l_c^2 (0.546B + 0.0000506)$$

19600 is the unit stress in the pier post; this assumes it to be of nickel steel of 1.4 times the strength of carbon steel, $\frac{l}{r} < 50$.

If we take the subdivided K system of web members we have

$$B_k = 3.75\left(\frac{a_{ti}}{U_{ti}} + \frac{a_{ci}}{U_{ci}}\right)\left(\frac{1}{\sin 2\beta} + \frac{1}{2 \tan \beta}\right)$$

The angle β and U_{ci} vary. The weights near the pier are the most important. The values at the distance $\frac{l_c}{4}$ from the pier give approximately correct results.

The height at the distance $\frac{l_c}{4}$ from the pier is $0.4028 l_c$. The

diagonals go over two panels in half the height. Taking 12 panels, we have

$$\tan\beta = \frac{\frac{1}{3}l_c}{0.4028l_c} = 0.8274$$

$$\beta = 39^\circ 36' \quad 2\beta = 79^\circ 12'$$

$$\sin 2\beta = 0.9823$$

$$B_k = 3.75\left(\frac{1.24}{28{,}000} + \frac{1.5}{17{,}000}\right) \times \left(\frac{1}{0.9823} + \frac{1}{1.6548}\right) = 0.000807$$

Therefore

$$W_{i \cdot web} = \mu l_c^2 (0.546 \times 0.000807 + 0.0000506)$$

$$W_{i \cdot web} = 0.000491\mu l_c^2$$

For parallel chords we have

$$W_{p \cdot web} = B_k \mu l_c^2$$

Taking the height which makes the weights of the chords equal with parallel and inclined chords, we have $h = 0.3441\ l_c$.

The diagonals of the subdivided K system go over half the height in two panels, therefore,

$$\tan\beta = \frac{0.3333}{0.3441} = 0.969$$

$$\beta = 44^\circ 6' \quad 2\beta = 88^\circ 12'$$

$$\sin 2\beta = 0.9995$$

$$B_k = 3.75\left(\frac{1.24}{28{,}000} + \frac{1.5}{17{,}000}\right)\left(\frac{1}{0.9995} + \frac{1}{1.938}\right)$$

$$B_k = 0.000754$$

$$W_{p\ web} = 0.000754\mu l_c^2$$

The difference in the weights of the webs for parallel and inclined chords is

$$\mu l_c^2 (0.000754 - 0.000491) = 0.000263\mu l_c^2$$

This result needs correction, because the formula for the weight of the web includes not only the main web members but

also the subsidiary web members, like suspenders, top-chord supports, and the subsidiary diagonals from the sub- to the main panel points.

These were allowed for by changing the coefficient 3.4 to 3.75. The weight of these subsidiary members remains substantially the same with both parallel and inclined chords.

For this reason the value $0.000491\mu l_c^2$ should be increased by $\mu l_c^2\left(0.000263 \times \frac{0.35}{3.75}\right) = 0.0000246\mu l_c^2$, which represents the reduction erroneously made by the formula in the weight of these subsidiary members. This gives for the correct weight

$$W_i \cdot_{web} = (0.000491 + 0.0000246)\ \mu l_c^2 = 0.000516\mu l_c^2$$

The weight of the chords is

$$\begin{aligned} W_i \cdot_{ch} &= 1.1665\mu l_c^2 \sec^2\delta \times 3.4\left(\frac{a_c}{U_c} + \frac{a_t}{U_t}\right) \\ &= 1.1665 \times 1.038 \times 3.4\left(\frac{1.45}{18{,}700} + \frac{1.25}{28{,}000}\right)\mu l_c^2 \\ &= 1.1665 \times 1.038 \times 0.0004154\mu l_c^2 \\ &= 0.000503\mu l_c^2 \end{aligned}$$

Eyebar top chords and nickel steel 1.4 times as strong as carbon steel were assumed. The combined weight of the chords and the webs is, with parallel chords,

$$\begin{aligned} W_p &= (0.000503 + 0.000754)\ \mu l_c^2 \\ &= 0.001257\mu l_c^2 \end{aligned}$$

For inclined chords,

$$W_i = (0.000503 + 0.000516)\ \mu l_c^2 = 0.001019\mu l_c^2$$

The ratio between the two weights is

$$\frac{W_i}{W_p} = \frac{0.001019}{0.001257} = 0.81$$

The weight of the cantilever trusses to carry the moving loads is, therefore, for inclined chords of the kind above described, 81 per cent. of the weight with parallel chords. This result is

very little modified if δ_c and δ_t are different without change in the heights. An increase of δ_t increases the weight of the pier post but reduces the sum of the weights of the chords.

When the load per foot is not uniform but increases toward the pier, the load at the end of the cantilever arm being P, the load per foot at the distance x from the end of the cantilever arm being $p + qx$, a loading similar to that from the wind, then

$$S_x = P + \frac{x}{2}(2p + qx)$$

$$M_x = Px + \frac{x^2}{6}(3p + qx)$$

With $h_x = a + bx$ the chord stress is

$$\frac{Px + \frac{x^2}{6}(3p + qx)\sec\delta}{a + bx}$$

The weight of the tension chord for the span length dx is

$$W_{i\cdot t\cdot ch} = \frac{3.4.a_t \sec^2\delta_t}{6U_t}\int_0^{lc}\frac{qx^3 + 3px^2 + 6Px}{a + bx}\,dx$$

$$qx^3 + 3px^2 + 6Px \div bx + a$$

$$= \frac{qx^2}{b} + \frac{3bp - aq}{b^2}\cdot x + \frac{6Pb^2 - 3abp + a^2q}{b^3}$$

$$- a\frac{(6Pb^2 - 3abp + a^2q)}{b^3(bx + a)}$$

Therefore $\displaystyle\int_0^{lc}\frac{qx^3 + 3px^2 + 6Px}{a + bx}\,dx$

$$= \frac{ql_c^3}{3b} + \frac{(3bp - aq)}{2b^2}\cdot l_c^2 + \frac{6(Pb^2 - 3abp + a^2q)}{6^3}\,l_c$$

$$- \frac{a(6Pb^2 - 3abp + a^2q)}{b^4}\,l_n\frac{a + bl_c}{a}$$

We have, therefore, for the weight of the inclined tension chord carrying the dead load,

$$W_{i\cdot t\cdot ch} = g_t \cdot \frac{1}{3}\left\{ \frac{ql_c^2}{3b} + \frac{3bp - aq}{2b^2} l_c^2 + \frac{6Pb^2 - 3abp + a^2q}{b^4} \left(blc - al_n \frac{hp}{a} \right) \right\}$$

$$\text{If } K = \frac{1}{3}\left\{ \frac{ql_c^2}{3b} + \frac{(3bp - aq)\, l_c^2}{2b^2} + \frac{6Pb^2 - 3abp + a^2q}{b^4} \times \left(bl_c - 2.3026a \log \frac{hp}{a} \right) \right\}$$

Then $W_{i\cdot t\cdot ch} = g_t K$

and $\dfrac{W_{\cdot i\cdot c\cdot ch} = g_c K}{W_{ich} = (g_t + g_c)\, K}$

where $g_t = \dfrac{3.4a_t \sec {}^2\delta_t}{2U_t} \cdot g_c = \dfrac{3.4a_c \sec {}^2\delta_c}{2U_c}$

The shear at the distance x from the end of the cantilever arm is

$$S_x = P + \frac{x}{2}(2p + qx)$$

This shear is partly in the chords. The shear in the two chords is

$$\frac{Px + \frac{x^2}{6}(3p + qx)}{a + bx} b$$

The shear in the web is, therefore,

$$S^x_{web} = S_x - S_{ch} = P + \frac{x}{2}(2p + qx) - \frac{(qx^3 + 3px^2 + 6Px)}{6\,(a + bx)} b$$

The shear area of the web is

$$\int_o^{lc} S^x_{web}\, dx = \int_o^{lc} \left\{ P + \frac{x}{2}(2p + qx) - \frac{(qx^3 + 3px^2 + 6Px)\, b}{6\,(a + bx)} \right\} dx$$

$$(bqx^3 + 3bpx^2 + 6bPx) \div (bx + a) = qx^2 + \frac{3bp - aq}{b} x$$

$$+\frac{6Pb^2-3abp+a^2q}{b^2}-\frac{a\,(6Pb^2-3abp+a^2q)}{b^3\,(a+bx)}$$

$$\int_o^{l_c} S^x_{web}\,dx = Pl_c+\frac{pl_c^2}{2}+\frac{ql_c^3}{6}$$

$$-\frac{1}{6}\left[\frac{q}{3}l_c^3+\frac{3bp-aq}{2b}l_c^2+\frac{6Pb^2-3abp+a^2q}{b^3}\times\left(bl_c-al_n\frac{h_p}{a}\right)\right]$$

The weight of the web is

$$W_{i\,web}=B\left[\int_o^{l_c} S^x_{web}\,dx\right]+\text{pier post.}$$

The pier post not included in the first part carries the vertical component of the top-chord stress to the pier.

For the top-chord stress at the pier we have the moment

$$M_{l_c}=Pl_c+\frac{l_c^2}{6}(3p+ql_c)$$

The vertical component of the stress is

$$\frac{\left\{Pl_c+\frac{l_c^2}{6}(3p+ql_c)\right\}\tan\delta_t}{h_p}$$

The weight of the post is

$$\frac{\left\{Pl_c+\frac{l_c^2}{6}(3p+ql_c\right\}a_{c_2}\tan\delta_t}{U_{c_2}}$$

We have, therefore,

$$W_{i\,web}=B\left\{\frac{l_c}{6}(6P+3pl_c+ql_c^2)-\frac{b}{2}K\right\}+\frac{l_c\,(6P+3pl_c+ql_c^2)\,a_{c_2}\tan\delta_t}{6U_{c_2}}$$

We are now prepared to find the ratio of the weights of the trusses for the dead loads, with inclined and with parallel chords.

We take for this purpose the same shape of trusses as we used in the comparison of the trusses for the moving and impact loads.

Taking $l_s = l_c = \frac{L}{3}$ and $P = \frac{pl_s}{2}$, also $a + bl_c = 4.5a = h_p$ and $p + ql_c = 4.5p$

$$a = \frac{l_c}{9} \text{ therefore } b = \frac{3.5}{9} = 0.3889\, q = \frac{3.5p}{l_c}$$

Expressing P, q and a by p and l_c we obtain

$$K = \frac{pl_c^2}{3}\left\{3 + \frac{9}{3.5} + \frac{8.5 \times 9^2}{3.5^3}\left(\frac{3.5}{9} - \frac{2.3026}{9}\log 4.5\right)\right\}$$

$$K = pl_c^2\left\{1 + \frac{3}{3.5} + \frac{25.5}{3.5^3}(3.5 - 2.3026 \log 4.5)\right\}$$

$$K = 3.0444\, pl_c^2$$

If $\delta_c = \delta_t = \delta$ then $g_c + g_t = \frac{3.4 \sec^2 \delta}{2}\left(\frac{a_c}{U_c} + \frac{a_t}{U_t}\right)$

If $U_t = 28{,}000$ $U_c = 18{,}700$

$$\tan \delta_t = \frac{b}{2} = \frac{1.75}{9} = 0.1944$$

$\tan^2 \delta_t = 0.038$ $\quad \sec^2 \delta_t = 1.038$

we obtain $a_c = 1.45$ $\quad a_t = 1.25$

$$g_c + g_t = 1.7 \times 1.038\left(\frac{1.45}{18{,}700} + \frac{1.25}{28{,}000}\right)$$

$$g_c + g_t = 0.0002156$$

$$W^d_{ich} = 0.0002156 \times 3.0444\, pl_c^2$$

$$= 0.0006564\, pl_c^2$$

For the weight of the web we have

$$W^d_{iweb} = B\left\{\frac{l_c}{6}(6P + 3pl_c + ql_c^2) - \frac{b}{2}K\right\}$$

$$+ \frac{l_c}{6}(6P + 3pl_c + ql^2)\frac{a_{c_2} \tan \delta_t}{U_{c_2}}$$

$$= 0.000807 \left\{ \frac{pl_c^2}{6} (3 + 3 + 3 \cdot 5) - 0.1944 \times 3.0444 pl_c^2 \right\}$$

$$+ 1.5833\, pl_c^2 \frac{1.75}{9} \times 1.5$$

$$\frac{}{19{,}600}$$

$$= 0.000824\, pl_c^2 \text{ subject to a slight correction.}$$

For parallel chords we have

$$M_x = Px + \frac{x^2}{6}(3p + qx)$$

$$S_x = P + \frac{x}{2}(2p + qx)$$

$$dW_p^d = (AM_x + BS_x)\, dx$$

$$= \left\{ APx + \frac{Ax^2}{6}(3p + qx) + BP + B\frac{x}{2}(2p + qx) \right\} dx$$

$$= \left\{ \frac{Aqx^3}{6} + (Ap + Bq)\frac{x^2}{2} + (AP + Bp)\, x + BP \right\} dx$$

$$W_p^d = \int_0^{l_c} dW_p^d = \frac{l_c^2}{24} \left\{ Aql_c^2 + (Ap + Bq)\, 4l_c + \right.$$

$$\left. 12\,(AP + Bp) + \frac{24BP}{l_c} \right\}$$

$$A = \frac{3.4}{206.4} \left(\frac{1.45}{18{,}700} + \frac{1.25}{28{,}000} \right) = 2013 \times 10^{-9}$$

$$B_k = 0.000754$$

Introducing in the equation for W_p^d

$$P = \frac{pl_c}{2}, \quad q = \frac{3.5p}{l_c}$$

we obtain, after reduction,

$$W_p^d = \frac{pl_c^2}{24}\,(13.5\, Al_c + 38B)$$

In this equation the part with the factor A gives the weight of

the chord, that with the factor B gives the weight of the web. We obtain for the web with K system

$$W_p^d web = \frac{pl_c^2}{24} \times 0.000754 \times 38 = 0.001194\ pl_c^2$$

We found for the web with inclined chords $0.000824\ pl_c^2$.
The difference is $0.000370\ pl_c^2$.
The required correction for the inclined web is

$$0.00037 \times \frac{35}{375}\ pl_c^2$$
$$= 0.000034 pl_c^2,$$

This gives for the corrected value of $W_{i.web}^{d}$ $= 0.000824$
$+ 0.000034$
0.000858

Adding the weight of the chords we obtain

$$W_i^d = pl_c^2\ (0.000858 + 0.000656)$$

or $W_i^d = 0.001514\ pl_c^2$

For W_p^d we find with $l_c = 600$ feet

$$W_p^d = \frac{pl_c^2}{24}\ (13.5 \times 2013 \times 10^{-9} \times 600 + 38 \times 0.000754)$$
$$= 0.001873\ pl_c^2$$

and $\frac{W_c^d}{W_p^d} = \frac{0.001514}{0.001873} = 0.808$

This ratio is, therefore, practically the same for loads uniformly distributed and for loads distributed like the dead loads.

The coefficient for cantilever arms of larger end height suitable for erection of the suspended span by building out from the ends of the cantilever arms were obtained in exactly the same manner.

All the foregoing formulas give either the weight of two trusses each with half the load or of one truss carrying the whole load.

SOME PRESENT-DAY METALLURGICAL PROBLEMS.*

BY

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METALLURGY is commonly defined as the art of extracting metals from their ores. If we look up the definition of the word "art," we find it stated that by art is meant the employment of means to the accomplishment of some end; a system of rules and established methods to facilitate the performance of certain actions; familiarity with such principles and skill in applying them to an end or purpose, as of a practical, useful, or technical character. If we apply this definition to metallurgy, we may say that metallurgy is the employment of chemistry to the accomplishment of extracting metals from their ores; in other words, metallurgy is applied chemistry, and that, by reason of a long period of application, a system of rules and established methods have been worked out in order to facilitate the performance of certain actions, or reactions, and that a metallurgist is one who is familiar with such methods and skilled in applying them to accomplish the end that he may desire.

Although metallurgy is applied chemistry, it is nevertheless true that metallurgy was the forerunner of chemistry, and is a very ancient art. As is evidenced by the large number of flint instruments which have been discovered from time to time, and in different localities, and which are now to be found in various museums throughout the world, there was a time when metals were entirely unknown, or else had been produced in such small quantities as to prohibit their general use. However, as time went on, men acquired the art of reducing the less refractory metals from their ores. Since copper was one of the first metals to be used, it may be that the copper came from what are known as native deposits, and that later the art of reducing it and other less refractory metals was discovered. Be that as it may, the

* Presented at the meeting of the Mining and Metallurgical Section, held Thursday, December 4, 1913.

copper, being soft, had to be hardened, and in ancient times, as to-day, this was done by the addition of a small proportion of iron, arsenic, or tin. At just what period the use of iron became general we do not know, for, due to the rapidity with which it oxidizes and likewise due to the fact that oxidation gradually proceeds through the whole mass, an iron instrument left lying alongside of a flint instrument would wholly disappear, and so it may be that iron was used much earlier than we have any record of. However, we are informed that explorers in Egypt and in Assyria discovered specimens of brass and iron which show that the Egyptians were acquainted with the use of iron as much as 4000 years ago, and possibly even before that time. In the early books of the Old Testament frequent references are made to the use of iron, and so, as before stated, we have every reason to believe that the use of iron was known in very early times. However, for centuries and centuries, metallurgy was more of a craft than an art, and new discoveries were for the most part the result of accident rather than of investigation. During the Middle Ages alchemy came into vogue, the principal aim of which, we are told, was to discover ways and means for the transmuting of base metals into gold. Although the alchemists were unsuccessful in their attempts, they nevertheless accumulated a large amount of facts on which was based the modern science of chemistry, but it was not until about a century ago that the later science was advanced to anything like its present state of development, and it has only been within that time that metallurgical processes have been carried out on a really scientific basis. As a matter of fact, the advances have been so rapid that the metallurgical profession has come to feel pretty well satisfied with itself. This is due to the fact that a large part of the advancement which has been made has been in the nature of large production at comparatively low costs.

I do not wish to be understood, in this connection, as inferring that present-day metallurgical practice has not been developed along perfectly normal lines; as a matter of fact, it has, but, now that it has reached its present state of development, the metallurgical profession should, and is, turning its attention to perfecting those processes which are in use at the present time, to devising others which will correct existing defects, and to solving some of the many problems which require a solution.

PROBLEMS CONNECTED WITH THE METALLURGY OF IRON AND STEEL.

Use of Inferior Grades of Ores.

Although we have heard more or less, during recent years, of the possibility of the world's iron ore supplies becoming exhausted, those who have made a special study of the subject inform us that they do not see any immediate prospect of this being the case. New ore fields are discovered almost every year, as, for example, those deposits in Brazil, a description of which was contributed to the International Geological Congress in 1910 by Dr. Derby, of the Brazilian Geological Survey, and which, according to Dr. C. K. Leith [1] and others, is greater than any deposit which has been discovered in the past, and whose ores are even of higher grade than those of the Lake Superior region. However, though it is true that the world's visible iron ore supply has been added to in this manner, it is equally true that the iron industry is beginning to feel the necessity of turning its attention to the treatment of ores which in the past have not been considered suitable for blast-furnace work, as, for example, those deposits which are found to the western end of the Mesabi Range, where the ore is interbedded with sand. These deposits are very extensive and for a long time they were not considered very valuable. They are now successfully concentrated, and so these deposits have added enormously to the value and extent of the Mesabi reserves. This is also true of many other deposits; namely, that, although too lean to work as raw ores, it is possible by concentration to bring the metallic content of the concentrates up to a point where they correspond to a fairly good ore. Although the metallurgical treatment of such ores requires special care and attention, due to the physical nature of the same, the smelting of them does not present any particularly difficult problems, as do those ores in which the iron is combined, either mechanically or chemically, with some other element or elements, and from which the iron is separated with difficulty, either by mechanical methods or in the metallurgical treatment of the same.

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[1] "Economic Geology," vol. vi, 1911, p. 670.

CLASSIFICATION OF IRON ORES.

The ores of iron which are used in the production of iron may be arranged according to the following general classification, as given by Turner:[2]

I. Magnetic oxide or magnetite.
   (*a*) Pure magnetites (FeO, Fe_2O_3 Fe_3O_4).
   (*b*) Magnetites in which part of the oxide of iron has been replaced by the oxide of another metal, *e.g.*, chromium (chromite), titanium (ilmenite), or zinc (franklinite).

II. Ferric oxide or hæmatite.
   (*a*) Anhydrous ferric oxide or red hæmatite (Fe_2O_3).
   (*b*) Hydrated ferric oxide, including brown hæmatite, limonite, bog ore, etc.

III. Ferrous carbonate or spathic ores.
   (*a*) Pure spathic ores.
   (*b*) Carbonate ores associated with clay (clay iron stones, argillaceous ores) or with bituminous matter (blackband).

As stated, these are the ores that are used in the production of iron. The majority of ores which have heretofore been treated in this country belong to II (*a*) more or less mixed with those of (*b*). For some time, iron ores of I (*a*), magnetites, were looked upon with disfavor, even when pure, as being difficult to reduce. The main objection to them, however, has been that they are so often mixed with other impurities, such as silica, titanium, and phosphorus. Silica, of course, if in excess, simply renders the iron too low in iron content to permit of profitable working; if too high in phosphorus, the iron will not make good steel, and if too high in titanium, the general impression has been that in smelting the ores in the blast furnace sticky slags were produced which interfered with the successful working of the furnace. As to the latter point, the same will be discussed farther along in this paper. On the whole, then, we may say, so far as the subject of ores is concerned, that the problem which immediately concerns the iron metallurgist is the treatment of the complex ores, so to speak,—that is, ores containing

[2] "The Metallurgy of Iron," by T. Turner, p. 50.

other metallic elements, and which are considered harmful for either of the following reasons:

(1) In the treatment of the ores in the blast furnace the other elements present in the ore may be reduced with the iron and so impart to it undesirable properties.

(2) As in the case of titanium ores, the titanium may render the treatment of the ores difficult, for reasons already explained.

As to the first of these problems, namely, the imparting to the iron of undesirable properties by an alloying element, it is now realized that certain elements which are present in iron ores impart beneficial rather than harmful effects to the fabricated metal. Take, for example, such ores as those of the Moa deposit in Cuba. They carry from 1.5 to 2 per cent. chromium, about half as much nickel, and 8.0 to 12 per cent. alumina. Although the treatment of such an ore is difficult in the blast furnace, and the steel maker is confronted with certain problems in converting the pig iron into steel, it is up to the metallurgist to conserve these two alloying elements, for alloy steels are now being used more and more, and in case of the Moa deposit it would seem as if there already existed in Nature a combination of elements in the same ore which have proved so beneficial in steel making. As stated by Dr. C. L. Parsons in Bulletin No. 47 of the U. S. Bureau of Mines, "the metallurgy of iron has reached a perfection beyond that of any other metal, and wastes have been eliminated to an extent that serves as some indication of what, perhaps, may be done in other industries when the same intelligent investigation has been applied." If this be true, then it would seem that progress in the metallurgy of iron and steel must be largely along the lines of conservation. However, it must not be inferred from such a statement that conservation has not been taken into account in the industry, for the saving and use of by-products testifies to that, as, for example, the making of a large part of the Portland cement of the country from blast furnace slags, etc. There is, however, still room for conservation, and especially along the lines of the conservation of those elements which are present in ores which have not been treated in the past, as there was no necessity of so doing, with the large supply of pure ores available, such as those of the Lake Superior region. It seems to the writer that one of the greatest problems in conservation, so far as the production of

iron is concerned, is the devising of some process whereby the iron which is associated with copper, lead, zinc, etc., in their ores may be saved instead of being thrown over the dump in the form of slag. Take, for example, the amount of iron that goes over the dump at Great Falls and Anaconda, Montana. At Great Falls it amounts to approximately 300 tons per day. At Anaconda it is about two and one-half or three times as much as this. At present this iron is lost, so far as its use as a metal is concerned. This is only one of the many plants of the country where the same thing takes place daily. We must, of course, admit that this iron plays its part in the smelting operations,—namely, as a basic neutralizing agent for the acid materials present in the charge,—and the problem, therefore, resolves itself into finding a suitable non-metallic base which will do the same work as the iron does, and a process whereby the iron may be saved and utilized. We must also admit, as has been pointed out by Mr. Wheeler, superintendent of the Boston and Montana Smelter at Great Falls, that, even if this iron could be recovered as metallic iron, it might be difficult to find a market for it in that region. However, that part of the difficulty would, no doubt, be overcome if a method could be found for recovering the iron.

If this iron should be alloyed with copper, perhaps all the better, as recent investigations tend to show that copper, when present in iron in proper proportions, inhibits corrosion. The writer is well aware that there are certain investigators who claim that such is not the case,—that is, that copper does not prevent corrosion, and that the purer iron is, the less liable it is to be corroded. With due deference to all those who hold this opinion, the writer believes that there is more evidence to warrant the belief that the presence of copper in proper amounts in steel does prevent corrosion, rather than that it does not, or, as some claim, by reason of its being an impurity, promotes it.

THE USE OF TITANIFEROUS ORES.

At the present time there is more or less prejudice among blast-furnace men against ores containing titanium oxide (TiO_2). Briefly stated, the history of the treatment of titanic ores is as follows: Numerous laboratory investigations have been conducted upon synthetic slags,[3] and small commercial blast furnaces

[3] By Rossi, Cox, Lennox, Stanley, Smolensky, and others.

have been operated upon titanium ores in Norway in 1850, at Norton, England, in 1868, and later by the McIntyre Furnaces in New York. In this connection it is well to note that all successful smelting of titanium iron ores has been done in small furnaces,—that is, furnaces having a daily capacity of from 3 to 15 tons of iron. The objections advanced by blast-furnace men of to-day against using titanium ores is that, when smelting ores containing even less than 1 per cent. of titanic acid, cyanonitride collects in the hearth and soon necessitates the closing down of the furnace. In no instance in the operation of the small furnaces was trouble reported from accretions of cyanonitride, and in no instance was it found impossible to make a fluid slag. At Norton, England, both coke and charcoal were employed at different times. Rossi used coke. As to why the small furnaces were not bothered with accretions of cyanonitride was probably due to the fact that the temperature was too low to form the carbide of titanium, which in combination with the nitrogen of the blast at the tuyères forms cyanonitride. The problem, then, of smelting these ores in the modern blast furnace is seemingly one of duplicating the temperature conditions of the small furnaces that have proved successful. On the other hand, there are metallurgists who state that it is quite easy and perfectly feasible to smelt titanium ores in the modern blast furnace, and operated in the same manner as when treating simple ores,—that is, the charge is made up with such a proportion of ores, fluxes, and fuel as to give a fluid slag,—and it is further stated that it is an ill-founded notion that titanic slags are viscous. Due to this diversity of opinion among iron blast-furnace men, the McIntyre Iron Company, which has large titaniferous ore holdings in New York State, has decided to carry on some experiments on a commercial scale at the plant of the Northern Iron Company at Port Henry, New York State. It is hoped, as a result of these experiments, to be able to dispel the uncertainties which now exist as regards the possibilities of being able to successfully smelt titanium ores. For this reason, it is expected that the experiments at Port Henry will be watched with a great deal of interest.

Thus we see that in the iron industry, so far as the use of raw materials is concerned, the problem of the metallurgist is:

(1) To so modify existing processes or invent new ones as

to enable him to successfully treat, from a commercial standpoint, those ores which have not hitherto been considered as suited to blast-furnace practice.

(2) To recover and make use of, as a source of iron, the vast amount of iron that is now thrown over the dump in the slags which are obtained in non-ferrous blast-furnace work.

(3) To prevent, if possible, the corrosion of the iron that is produced, either by the addition to it of some element which will inhibit corrosion, or by the production of a pure metal, which, as before stated, is the way some claim to be able to prevent corrosion. One of the greatest advances which could be made in conservation would be the discovery of some means whereby the corrosion of iron and steel could be prevented. Not only would this mean conservation of iron but of coal. If we calculate the amount of fuel that is used for all purposes, including transportation, installation, etc., we find that it amounts to about three tons of coal for every ton of pig iron produced. In collecting data on this subject, Dr. G. A. Hulett, Chief Chemist of the Bureau of Mines, plotted the data, and drew a curve, representing the production of pig iron in every five years since 1880. Multiplying these values by three, we get the tons of coal used in making this iron. The rate of production of pig iron in the United States for the last 25 years gives a steep curve, but the rate of increase of coal utilized for this purpose gives a curve three times as steep, and the results are most striking. The following table by Dr. Hulett indicates the situation:

TABLE I.

Date.	Million metric tons of pig iron.	Coal.
1880	3.2	9.6
1885	4.6	13.8
1890	6.8	20.4
1895	10.0	30.0
1900	14.0	42.0
1905	19.4	58.2
1910	26.0	78.0
1915	33.5	100.0
1920	41.5	124.5

The facts and curves presented by Dr. Hulett show the very great importance, even from this standpoint, of doing work

which will prevent the corrosion of iron and steel. As pointed out by him, no consideration has been given to the amount of labor used in producing the iron and steel, or interest on the capital invested in these operations; so that this is only one phase of the importance of the question of corrosion.

As regards conservation, although the amount of iron and steel which has been produced each year has steadily increased, less ore is now required to produce a ton of steel than was the case, say, in 1890, when nearly four tons of ore were consumed per ton of steel produced, whereas ten years later only 2.7 tons were required, and in 1910 only 2.1 tons. As pointed out by Chamberlain,[4] though several factors are involved, this decrease in the amount of ore used has been due to the growth of the open-hearth process and the consequent utilization of scrap.

FINE ORES.

As is well known, it is only within recent years that this problem has arisen in iron blast-furnace work, for so long as only Lake Superior ores were used there was not much trouble from this source. However, as the richer ores began to grow scarcer, the poorer ones began to be concentrated, until now it is stated that there are over a million tons thus produced annually in the State of New York alone by magnetic concentration. Meanwhile, as before stated, the concentration by washing of the lean and sandy ores of the Mesabi range has increased until the amount of concentrates so produced is something over three million tons annually. Naturally the treatment of these concentrates in the blast furnace has greatly increased the percentage of fines in the burdening of the furnace, with the result that the furnaces are more difficult to operate than when treating coarser ores. Such being the case, it would seem that some preliminary treatment of these fines, for the purpose of agglomerating them, as well as the flue dust which is produced, would tend to increase the efficiency of the blast furnace. Inasmuch as considerable progress has already been made along this line,—that is, in the way of nodulizing and sintering fine ores,—it may not be strictly correct to speak of the treatment of fine ores in this manner as

[4] G. D. Chamberlain, "Some Developments of the Iron and Steel Industry," *Journal of Industrial and Engineering Chemistry*, vol. v, May, 1913.

a metallurgical problem. Be that as it may, there is, no doubt, considerable room for study along these lines, especially as regards the physical characteristics and the chemical composition of the agglomerated or sintered material, and the effect which the use of these materials has on the efficiency of the furnace.

PROBLEMS CONNECTED WITH THE OPERATION OF AN IRON BLAST FURNACE.

Turning now to the operation of the blast furnace, although, as previously stated, the metallurgy of iron has reached a perfection beyond that of any other metal, there are still many problems to be solved in connection with the operation of the same, many of which are mechanical as well as metallurgical. For example, Mr. James Gayley discovered and proved some eight years ago that the use of a dry blast in blast-furnace work greatly increased the efficiency of the furnace. Although, so far as the writer is aware, it is not doubted that the use of the dry blast will greatly increase the efficiency of a furnace, nevertheless its use has not become so general as was predicted, and this, I should say, is due to the fact that the cost of installation of the Gayley system is quite heavy. If this be true,—namely, that the use of the dry blast does increase the efficiency of the furnace, but that its adoption has not become general on account of the cost of installing it, or for some other reason,—then the problem resolves itself into that of being able to accomplish the same results by some other means. In addition to the problem of the use of the dry blast, there are many others, which we will not attempt to discuss fully, but which may be stated as follows:

(*a*) Production of uniform grades,—*i.e.*, less of off-grade iron.

(*b*) Economy in coke,—*i.e.*, by making coke that is put into the furnace do more work, either in the smelting process itself, or by increasing the efficiency of the hot blast stove, or by some other means.

Some interesting work has recently been done along this line at a blast furnace in Belgium. Not only do they dry the blast, but they have also increased its oxygen content from 20.8 per cent. to 23.0 per cent. The oxygen is obtained from liquid air. After separating the oxygen from the more volatile gases,

the oxygen is finally compressed in holders and used as desired. The enrichment of the blast in this manner is not a new idea. J. E. Johnson obtained patents on a process of this kind in 1908, and has since given the matter more or less thought.

(*c*) A further utilization of the slag.

As is well known, iron blast-furnace slags are now used in making a large proportion of the Portland cement which is manufactured in this country. However, there is still a large part of it which is put to no better use than ballast for railroads, or for road-making, for which some other material can just as well be used, and so there is still room for a further study of the possible use of iron blast-furnace slags.

THE USE OF THE ELECTRIC FURNACE IN THE PRODUCTION OF IRON FROM ITS ORES.

As is well known, the electric furnace was not evolved in the hopes of competing with the blast furnace, but of finding a furnace and a process which would be able to produce iron in those localities where blast-furnace practice was not feasible, or, as in Sweden, where the cost of blast-furnace fuel was becoming so great as to cause the existing blast-furnace practice to become prohibitive.

DIFFICULTIES THAT HAD TO BE OVERCOME.

The idea of those who first took up the work was to construct a shaft similar to a blast-furnace shaft, and to then substitute electrodes for tuyères. When this was tried out in practice it was found that the life of the furnace wall in the neighborhood of the electrodes was short-lived, and no amount of water-cooling served to obviate this difficulty, but rather increased it if anything, due to jackets burning out, etc. As was early observed by Heroult, the proper way to maintain the walls of an electric furnace crucible is to get the electrodes as far removed from the side walls as possible, and in the development of the electric iron reduction furnace in California and in Sweden it was found not only necessary to do this but also to keep the charge as far as possible from the roof of the crucible, for unless this was done the roof was short-lived, due to the intense local heat which is generated at the point where the electrodes enter the charge. The maintenance of the roof of the crucible was one of the most

serious difficulties that had to be overcome, and in overcoming this difficulty the present shape of the roof of the crucible and the manner in which the electrodes are introduced into the crucible were evolved. As a further protection to the roof of the crucible a part of the gases escaping from the top of the shaft were returned to the crucible in order to cool the same, as well as to assist in the reduction of the charge.

THE ELECTRODE PROBLEM.

Although the Swedish experimenters did not have so much to contend with in this respect, in California the difficulty in securing electrodes that would meet the requirements greatly interfered with the progress of the work. When the latter was begun in California the manufacturers of electrodes in this country had not previously been called upon to furnish electrodes of such large cross section, namely, about 20 inches square and about 72 inches in length. Of those first furnished, that part of the electrode projecting into the crucible would either break off completely after it became heated up, or else large chunks would spall off from it, and, as can be readily imagined, these broken pieces gave no end of trouble in operating the furnace. That others also encountered this difficulty is evident from the following quotation from a paper presented by William R. Walker at a meeting of the American Iron and Steel Institute in 1912:

"Our problems, mechanical, metallurgical, and otherwise, proved many, and our experience soon demonstrated that the conditions surrounding the successful operation of a large electric furnace were in many respects entirely different from those involved in the use of smaller units. In illustration, the demands of a 15-ton electric furnace proved to be far in advance of the art of manufacturing electrodes. Our necessities represented a requirement that the electrode manufacturers of America and Europe had not been called upon to meet, and it took much time and money before there was finally accomplished the 20-inch round amorphous carbon electrode, and that is now being used at South Chicago."

At the plant of the Noble Electric Steel Company it was finally decided to try graphite electrodes. These worked very satisfactorily in all respects except for the fact that on account of the angle at which it was necessary to insert them in the cru-

cible, due to the manner in which the furnace was constructed, they were subjected to a severe strain which caused them to break at the threaded joints. The electrode problem is, however, no longer a serious matter. As stated by Mr. Walker in his paper, the 20-inch round electrodes now in use at South Chicago give satisfaction, and at Trollhättan the large carbon electrodes of about the same size as those used at South Chicago meet the requirements.

PROBLEMS YET TO BE SOLVED IN CONNECTION WITH IRON ELECTRIC FURNACE WORK.

Reduction in Shaft of Furnace.

In an ordinary blast furnace the weight of the gases produced exceeds the weight of the charge by 30 to 50 per cent., whereas in electric reduction furnace work the gases evolved amount to only about 40 per cent. by weight of the charge. In other words, in the blast furnace in the production of one ton of iron there is three to four times as much gas given off as there is in the production of one ton of iron in the electric furnace. Moreover, the temperature of the gas as it leaves the tuyères may be as much as 1600° C., whereas the highest temperature stated to have been attained at the lower end of the stack in the work at Trollhättan is 985° C. Even if this be granted, and that the temperature of the gas as it enters the stack may have as high a temperature as 1000° C., as pointed out by Frick, " inasmuch as the weight of the gases produced is only 35 to 45 per cent. of the weight of the charge, the charge would not be heated to more than about 350° C., due to the fact that the specific heat of the gas and that of the charge are about the same." Thus we see that we cannot expect complete reduction in the shaft if we depend solely upon the heat from the gases which are generated in the regular manner. In other words, the shaft acts merely as a preheater, with possibly a small amount of reduction taking place therein, but for the most part reduction takes place in the crucible by means of solid carbon.

Thus we see that an electric furnace may be operated in one of two ways:

(1) Reduction of the ore in the crucible by means of solid carbon, with no attempt at reduction in the stack, as is done in California at the present time by the Noble Electric Steel Company.

(2) Partial or complete reduction in the stack, this being the idea on which the Swedish furnaces are operated.

As we all know, that which may theoretically be best is not always so in practice. In this instance we are not able to judge by results, as we have not enough accumulated data at our disposal, and so can look at the matter only from a theoretical standpoint.

Theoretically,[5] reduction should take place in the shaft of the electric iron reduction furnace. The problem, therefore, that presents itself is how best to accomplish this; that is, to so operate the furnace as to cause the oxide of iron to be reduced by the time it reaches the electrodes, so that the electric energy shall be called upon to furnish only enough heat to melt the sponge iron, to scorify the gangue materials and fluxes, and to furnish to the products of the furnace the heat necessary for their separation and removal from their furnace.

Among other problems that yet remain to be solved in connection with electric iron furnace work may be mentioned:

1. *The Size of the Unit.*—The electric reduction furnaces now in operation vary in size, as regards their horsepower, from 1500 up to 3500, while the largest yet designed by A. B. Elektrometall (of Sweden) is 7500 kilowatts. As can be readily understood, it is quite important that the size of the unit be made as large as possible, and the writer is of the opinion that no one is prepared to say at the present time just what that size will ultimately be.

2. *Efficiency of the Furnace.*—The efficiency of the furnace will also be increased as time goes on, and probably by improvements along the following lines:

(*a*) The utilization of the waste gases.
(*b*) The securing of a high-power factor.
(*c*) The correction of induction losses.
(*d*) The further study of the single-phase furnace *vs.* the three-phase furnace.

As regards the latter, inasmuch as practically all large power installations are three-phase, it would seem that the only logical thing to do is to use three-phase current in electric furnace work,

[5] A theoretical discussion of this subject is given by the author in a paper, "Reduction of Iron Ores in the Electric Furnace," original communication to Eighth International Congress of Applied Chemistry, vol. xxvi, p. 519.

but from data submitted by Catani [6] it would seem as if the monophase furnace is more efficient that the polyphase furnace.

3. *The Use of Coke and Crude Oil as Reducing Agents.*—As is well known, of the coke charged into an iron blast furnace, only about two-thirds of the same is used for producing the heat necessary for carrying on the process, while the other one-third is used as a reducing agent. Therefore, if the electric furnace is used for the smelting of iron ores, only enough carbon has to be supplied to unite with the oxygen of the ore; in other words, to reduce it. So far only coke and charcoal have been tried to any extent as reducing agents in the reduction of iron ores in the electric furnace, and of these only charcoal has proven satisfactory. This has been shown by repeated trial runs at Trollhättan, and has been forcibly demonstrated by the fact that the plant at Hardanger, Norway, where coke was used as a reducing agent, after being in operation for about nine months, was forced to close down, due to the fact that nowhere near the same amount of pig iron could be produced per killowatt-year of electrical energy expended as can be produced when charcoal is used. That this is true is doubtless due to the fact that coke is a much better conductor of electricity than is charcoal, especially after it becomes hot. For this reason, when using coke, the resistance of the charge becomes lowered, and, as the smelting in the electric iron reduction furnace is done by the heat produced by the resistance which the electric current meets with in passing through the charge between the electrodes, more electrical energy is required to produce the same amount of heat. Likewise, as stated by Crawford,[7] the electrical conductivity of coke "is so good that much of the current passes between the electrodes in the upper part of the furnace. The smelting zone is thereby raised and the furnace runs hot on top with attendant melting of the arches (of the roof of the crucible) and cold at the bottom.

"*Second.*—The coke, because of its density and high crushing strain, does not break down like charcoal as the burden descends, hence less surfaces of carbon are exposed to be oxidized by the ore and there is a less intimate mixture of the two.[8] Re-

[6] *Trans. Amer. Electrochem. Soc.*, vol. xv, p. 168.

[7] "Progress of Electric Smelting at Heroult, California," by John Crawford, *Metallurgical and Chemical Engineer*, July, 1913, p. 386.

[8] This intimate mixture is, of course, very necessary where the reduction is almost entirely performed by solid carbon, as is the case in the practice at Heroult, California.—THE AUTHOR.

duction of the ore takes place more slowly, the silicon in the iron is lowered, the consumption per ton increases, and the efficiency of the furnace is reduced."

Inasmuch as either charcoal or coke is the only practical reducing agent we know of at the present time,—that is, for use on a large scale,—and as it would seem, from what has just been stated, that it is impracticable to use coke, this necessarily limits the electric iron reduction furnace to the use of charcoal. If this be true (and it undoubtedly is at the present time), it is unfortunate, especially as regards the situation in Southern California and in other of our Western States, where there are quite large iron deposits of A No. 1 ore, but where coke is too expensive to permit of ordinary blast-furnace smelting, and where the use of charcoal for electric-furnace work is entirely out of the question, as there are no forests to furnish the wood necessary for its production. On the other hand, crude oil is generally more or less plentiful and comparatively cheap in such districts, especially in California. Aside from its possible use in connection with electric-furnace work, those interested in the subject have for years been considering the possibility of using crude oil as a reducing agent. So far as we are aware, this has not as yet been successfully done. That the carbon and the hydrogen of the oil will reduce iron oxides is self-evident, but as yet no one seems to have been able to solve the problem as how to bring the ore and the oil together at the proper temperature. Perhaps ultimately a suitable process may be devised whereby the oil may be used as a reducing agent, and thus broaden the field for the possible application of the electric furnace in the reduction of iron ores, for, from what has been stated above, it is apparent that the electric furnace is limited for such purposes at the present time to those localities where charcoal and electric power are comparatively cheap.

STEEL.

Turning now to the production of steel, one of the most serious problems which presents itself is the fuel problem. As is well known, there has been a gradual transition from the Bessemer to the open-hearth process, till to-day the Bessemer process seems destined to act simply as an auxiliary to the open-hearth

and electric-furnace process. As is also well known, whereas the heat in Bessemerizing is obtained by the rapid oxidation of silicon and carbon content of the metal, in the open-hearth the heat is derived from an extraneous source; that is, from the combustion of coal or of gas, and the latter may be either producer or natural gas. When coal has been used either directly or for producing gas, the sulphur content of the same has been less than 1 per cent. Nowadays, however, coals with as low a sulphur content as this are becoming scarce, as is, likewise, natural gas. When a coal, or the producer gas made from it, is used which contains a larger sulphur content, the sulphur content of the steel is appreciably increased. The problem, therefore, that presents itself is how these high-sulphur coals may be used without impairing the quality of the steel.

Use of "Powdered" Coal and Loss of Sensible Heat.

In steel making there is still room for considerable improvement along the line of heat efficiency; that is, of getting out of the fuel that is used in heating the furnace the greatest number of calories and of conserving or utilizing the heat after it has been obtained. As a means to this end, a great deal of work is at present being done, and has been done, on the use of powdered coal as a fuel. By using coal in this manner the producer can be done away with. Although the prospects for being able to use powdered coal successfully are very favorable, much work will yet have to be done before a method has been perfected, and even when solved the problem of stack losses in sensible heat, which are enormous in the open-hearth work, will still have to be dealt with. At present much effort is being directed toward saving this heat by using it under boilers, etc.

The Use of the Electric Furnace as a Solution of These Problems.

As is well known, in the electric furnace the heat is generated at the point where it is needed. Take, for example, the Heroult furnace, or any other furnace of that type. The furnace itself is nothing more than a chamber built up of refractories and encased in steel. Through the roof of the chamber pass the electrodes, two or three in number, as the case may be, and the heat is generated in the furnace at the point where these electrodes

come in contact with the charge. Thus it is plain to be seen that by using the electric furnace the two difficulties above mentioned may be avoided; that is, contamination of the steel by the impurities contained in the fuel, and the loss of heat which results in an open-hearth practice by reason of the large volume of gas that is constantly passing through the furnace and carries with it a large amount of sensible heat which is not conserved in the checker work of the regenerators. It would seem, therefore, as if the electric furnace were the solution of this problem, for not only would the difficulties above mentioned be avoided, but a better grade of steel would be produced. As a matter of fact, a better grade of steel can be produced in the electric furnace than in the open-hearth furnace. In corroboration of this statement may be cited the experiments which during the past four years have been conducted by the United States Steel Corporation.[9] These experiments have shown that the electric-furnace process, as compared to Bessemer and open-hearth processes, has the following advantages:

(1) The more complete removal of oxygen.

(2) The absence of oxides caused by the additions, such as silicon, manganese, etc.

(3) The production of electric steel ingots of 8 tons weight and smaller that are practically free from segregation.

(4) Reduction of sulphur to 0.005 per cent. if desired.

(5) Reduction of phosphorus to 0.005 per cent., as in the basic open-hearth process, but with the complete removal of oxygen.

Some test rails made from acid Bessemer steel refined in a basic electric furnace were comparatively soft, but have shown superior wearing qualities compared with Bessemer rails in the same tracks and under the same service conditions. The steel from which these rails were made has the following composition: Carbon, .055 per cent.; manganese, 0.137 per cent.; silicon, 0.13 per cent.; sulphur, 0.017 per cent.; and phosphorus, 0.022 per cent. It is stated that there are now about 5600 tons of standard electric steel rails which have been in service in the United States for the past two years, and that these rails have been subjected to all sorts of weather and other conditions. It is also stated that no basic electric steel rails in service in this country

[9] *Metallurgical and Chemical Engineering*, vol. x, 1912, p. 371.

have yet broken. The results of a number of tests have shown that electric steel of a given tensile strength has a slightly greater amount of elongation than basic open-hearth steel, and is somewhat denser than basic open-hearth and acid Bessemer steel. The result of some tests made at South Chicago by the Illinois Steel Company (see Table II) shows 15.5 per cent. increased ultimate strength for electric steel and 11.3 per cent. decreased elongation (in 2-inch) as compared with open-hearth plate steel of approximately the same chemical analysis.

TABLE II.[10]

Statement of Averages of Ultimate Strength and Per Cent. Elongation of Electric and Open-Hearth Plate Steels.

Carbon, per cent.	Ultimate strength, lbs. sq. in.	Per cent. elongation.	Carbon, per cent.	Ultimate strength, lbs. sq. in.	Per cent. elongation.
0.08	59,194	27.25	0.08	51,690	32.00
0.12	64,080	26.05	0.12	56,510	29.70
0.16	69,220	25.25	0.16	52,901	28.61
0.20	72,853	22.82	0.20	58,294	28.82
0.24	69,540	23.12	0.24	63,560	26.25

Why is it, then, that the electric furnace is not more extensively used for steel making? It seems to be a matter of cost. In other words, where large tonnages are required, due to the comparatively high cost of electric power, steel can be more cheaply produced in the Bessemer or open-hearth furnace than in the electric furnace. For this reason, unless a better grade of steel is derived than can be produced in the Bessemer or open-hearth processes, the latter processes are used.

Cost of Production of Steel in the Electric Furnace.

This will, of course, depend upon local conditions. As a rule, the cost of power is the governing factor in electric-furnace work; but in the refining of steel this item of cost does not play such an important part as it does in some other electrometallurgical processes. For example, electric-furnace plants are being successfully operated in localities where power costs ten cents

[10] *Transactions American Electro-Chemical Society*, vol. xix, 1911, p. 221.

per kilowatt-hour, but when the cost for raw material would be the same for either the open-hearth or for the electric furnace. On the other hand, a plant may be located in a remote locality where the cost of power is very low, say 0.20 cent per kilowatt-hour, but when the cost of raw materials may be comparatively high. In Tables III–VIII are given the estimated cost of producing steel at various places and in various types of furnaces.

These figures are only for conditions as prevailing at Ugine.

TABLE III.

Production Cost of Steel from Cold Scrap, Girod [11] Furnace, Ugine, France.

	2.5- to 3-ton furnace.		10- to 12-ton furnace.	
Raw materials:				
Scrap, 1100 kilogrammes, at $15 per ton	$16.50		$16.50	
Slag	0.46		0.46	
Deoxidizing additions and recarburizing	0.70	$17.66	0.70	$17.66
Producing cost:				
Electric power, 850 and 750 kilowatt-hours, at 0.20 cent	3.40		3.00	
Electrodes at $64 per ton	0.60		0.70	
Wages	0.60		0.30	
Maintenance and repairs	2.40	7.00	1.60	5.60
Total cost per ton		$24.66		$23.26

TABLE IV.

Production Cost of Steel from Molten Charge, Girod [11] Furnace, Ugine, France.

	2.5- to 3-ton furnace.		10- to 12-ton furnace.	
Raw materials:				
Liquid steel, 4 per cent. loss in heating (1040 kilogrammes, at $16 per ton)	$16.64		$16.64	
Slag	0.40	$17.04	0.40	$17.04
Producing cost:				
Electric power, 275 and 300 kilowatt-hours, at 0.2 cent	1.10		0.80	
Electrodes at $64 per ton	0.25		0.25	
Wages, 8 heats per 24 hours	0.20		0.20	
Maintenance and repairs	0.80	2.35	0.50	1.75
		$19.39		$18.79

[11] *Metallurgical and Chemical Engineering*, vol. x, 1912, p. 663.

NOTE.—The estimate of Girod does not include expense for ingots, moulds, superintendence, laboratory, amortization, general charges, or royalty charge.

TABLE V.

Refining Cost Per Ton of Steel from Molten Charge, in a 2-Ton Gronwall Furnace Operating under English Conditions, Sheffield, England.

	Without dephosphorizing.	Dephosphorizing.
Number of charges per week	40	30
Tons per week	80	60
Cost per ton of steel:		
Power at 0.5 cent per kilowatt-hour	200 kw.h. $1.00	250 kw.h. $1.25
Repair of roof	0.16	0.20
Dolomite (100 pounds per charge)	0.17	0.17
Electrodes at 4 cents per pound	0.18	0.22
Sundry costs	0.14	0.16
Royalty	1.50	1.50
	$3.15	$3.50

In the above estimate the cost of labor, amortization, interest, raw materials, and general charges are omitted. The figures refer to the production of steel with sulphur and phosphorus below 0.02 per cent.

TABLE VI.

Cost of Production for a 2-Ton Three-Phase Rochling-Rodenhauser Furnace Producing Steel for Castings from Molten Metal.

	Per ton of steel.
Lining: Raw materials	$0.304
Wages	0.043
Heating up furnace: 2000 kilowatt-hours, at 2.17 cents for 160 tons	0.272
Refining: 300 kilowatt-hours per ton, at 2.17 cents	6.52
Wages	0.425
Materials: 20 kilogrammes mill scale	0.082
30 kilogrammes lime	0.087
6.5 kilogrammes ferrosilicon	0.485
4.0 kilogrammes ferromanganese	0.214
Power for cooling arrangements	0.196
Repairs	0.194
	$2.822

To the above must be added interest and depreciation of the plant. Taking 5 per cent. interest and 10 per cent. depreciation on $13,380, which will be the approximate cost of the plant, gives $2000 per year, which for 300 working days would give 41.2 cents per ton, so that the conversion of the fluid product from the basic converter into high-grade steel suitable to replacing crucible steel for steel castings will cost $9.23 per ton, exclusive of cost of molten steel charged: this on a basis of three men operating the furnace.

TABLE VII.

Cost of Producing Rail Steel in a 7-Ton Three-Phase Rochling-Rodenhauser Furnace from Fluid Steel.

	Per ton.
Raw materials, steel at $15.54 and all fluxes	$16.56
Power at 0.58 cent per kilowatt-hour: Heating up	0.02
Refining	0.72
Wages: On furnace	0.16
On lining	0.008
Lining materials	0.072
Tools	0.097
Repairs	0.194
Amortization and interest	0.168
Licenses	0.582
	$18.62

This estimate includes all items and, aside from the cost of molten steel, gives $3.08 per ton as the refining cost with power at 0.58 cent per kilowatt-hour.

TABLE VIII.

Cost of Producing Steel in the Rochling-Rodenhauser Furnace from Cold Scrap, at Lansdowne, Pa.

Raw material (scrap)	$14.50	
Oxidation loss about 2 per cent	.29	
	$14.79	$14.79
Current, 844 kilowatt-hours, at 0.7 cent	5.91	
Fluxes	.76	
Ore	.05	
Labor	1.50	
Tools, repairs and bottom	.76	
Cooling air for transformers	.12	
Conversion cost	9.10	9.10
Interest, 5 per cent.; depreciation, 10 per cent. per ton		1.50
Cost of one gross ton (2240 pounds) electric steel ready to pour		$25.39

Thus we see that under present conditions the electric steel furnace cannot compete, so far as costs are concerned, with the Bessemer or open-hearth processes. However, there is now a tendency among consumers of steel in larger amounts for rail and structural purposes to require a higher grade steel at an increased cost rather than a steel of the grade of the acid Bessemer, or even of basic open-hearth steel at a lower price.

The Field for the Electric Furnace.

It is probably in combination with either the acid Bessemer or the basic open-hearth processes that the electric furnace seems destined to occupy a prominent position in the manufacture of tonnage steel. The cost of super-refining the molten steel from either of these processes in the electric furnace, exclusive of cost of molten steel, varies from $1.50 to $2.25 per ton, depending on the cost of power and the impurities to be removed.

Piping in the Ingot.

One of the problems which has confronted the steel makers from the very beginning has been that of the ever-present pipe in the ingot, and it is still a problem. Various remedies for this difficulty have been proposed from time to time, such as keeping the ingot open by means of a hot top so that the top does not solidify before the remainder of the ingot has had a chance to do so. Others have proposed fluid compression, but it has never developed into general application. Recently Talbot has come forward with a method, which had been proposed some years ago, of producing a sound ingot by lateral compression of the ingot while its centre is still liquid.

Talbot states [12] that his investigation of the subject has extended over several years, that his results and conclusions are based upon an investigation of more than one hundred large ingots, and that he has no hesitation in saying that any one following with ordinary care the method he has outlined will be able to obtain similar results on ingots of approximately the same size. Be that as it may, most steel metallurgists feel that none of the methods proposed exactly meets the requirements, and that heavy cropping is the surest and safest remedy where the pipe seriously affects the products.

[12] B. Talbot, "The Production of Sound Steel by Lateral Compression of the Ingot," *The Journal of the Iron and Steel Institute*, vol. 87, p. 30.

THE DIRECT REDUCTION OF IRON ORES.

The term "direct process" in relation to iron smelting has been used with two distinct meanings. It has been applied to the methods of making wrought iron and steel without overcarbonizing the product at any stage in the process, to distinguish such a process from the usual procedure of first making pig iron in the blast furnace and subsequently decarburizing and refining it. The phrase "direct process" has been applied also to methods of smelting iron in distinction to the blast-furnace practice of burning a great part or all of the carbon to carbon monoxide at the tuyères, thereby proportionally lessening the power possessed by the carbon of abstracting oxygen from the ore. In this latter sense, therefore, the distinguishing feature of a direct reduction process is that, by dispensing with the blast, the oxidizing of any of the reducing material by any other agent than the oxygen of the ore is avoided.

Using the term with this meaning, the simplest "direct reduction" method is obviously to place ore and coal (or other reducing substance) in a closed vessel and apply heat externally. At the proper temperature, as is well known, the carbon will take the oxygen from the ore and release the metal. The hitherto insurmountable difficulty in applying the seemingly very simple principle on an industrial scale to iron ore reduction has been on the score of heat economy. The advantages which such a process offers have attracted iron manufacturers for generations, and to-day, with the advent of the electric furnace, the hope and effort of metallurgists are again turned to the solution of this problem, and with much brighter prospects of success than have cheered the experimenters of the past.

The advantages of a practicable direct process are many. The blast furnace as it is at present constructed and operated is clearly restricted to the use of charcoal, coke, and anthracite. Its product, if used for steel making, contains carbon much in excess of the quantity needed. This excess carbon represents a waste of fuel and needlessly lengthens the refining operation. The nitrogen of the blast, by diluting the reducing gases of the blast furnace and by increasing their volume and consequently their velocity, does not permit them to remain in contact with the ore a sufficient length of time to exercise fully their reducing power.

A direct reduction process, on the other hand, holds forth the promise of utilizing fuel and reducing material of most diverse character, such as soft and non-coking coals, peat, oil, natural gas, etc. In many localities there are extensive deposits of readily minable ore of good quality, which are unavailable, because fuel suitable for blast-furnace operation can be obtained only at prohibitive cost. In new countries, where small quantities of material are required, a direct process capable of operation in much smaller units than a blast-furnace installation would be invaluable.

The possibility of reducing iron directly from its ore, and even of reducing and refining the metal in a single operation, is not questioned. That such procedure is more rational than the present general method of making an impure pig iron and subjecting it to a refining process to convert it into steel is questioned only in its economic aspects. Neglecting, for the present, electrical methods, direct processes have hitherto proceeded along one of two lines. The oldest method, typified by the native or Catalan forge, is to heat the mixture of ore and fuel in an open hearth. The second method is to heat the mixture of ore and fuel in a closed vessel from without.

As previously stated, there is abundant evidence that wrought iron and steel were in use in many parts of the world many centuries before the Christian era, and there is little doubt but that they were made by such simple and direct processes as are still used by natives in parts of India, Bermuda, and Africa.

Modified types of this old process have come down to us even to the present day. In this country it was the American bloomery. The yield of metal from such a furnace was relatively small and the cost of labor and fuel and the loss of iron in the slag relatively high. Coke can not be used as a fuel except in a few special cases, because the spongy iron produced at the low temperature readily absorbs sulphur from the charge. So far as the writer is aware, there have been no bloomeries in operation in this country since 1901.

Turning now to the second method, namely, the heating of the mixture of the ore and fuel together, the following are some of those processes that have been devised in the past for the purpose of producing iron and steel directly in this manner.

One of the earliest of the gas-fired open-hearth furnaces for

direct reduction was introduced by Sir W. Siemens in 1873.[13] A mixture of fine ore and coal was treated in a rotating cylindrical furnace heated internally by producer gas. A very basic and fluid slag was obtained with the almost complete removal of phosphorus and sulphur. The resulting metal was balled and squeezed and used either in the open-hearth steel furnace or for working into wrought iron. The deoxidation of the ore was successful, but the reoxidation of the sponge and loss of iron in the slag were fatal to the process.

Many modifications of the reverberatory hearth have been tried. In the Renton Process (1851) a mixture of fine ore and coal or charcoal in vertical retorts erected at the end of a puddling furnace was heated by the waste gases and partially reduced before it was dropped to the puddling hearth. Wilson's process,[14] which was tried in this country in the '80's and proved commercially unsuccessful, did not differ essentially from Renton's process. Several furnaces combined a reverberatory furnace and a shaft. Into the latter the ore mixture was charged and heated by direct contact with the hearth gases with the object of further utilizing the reducing power of the gases and obtaining preliminary partial reduction.

The Eames process,[15] as tried by The Carbon Iron Company of Pittsburgh, consisted in reducing very fine ore mixed with coke or "graphite anthracite" on the bed of a reverberatory furnace fired with natural gas. The charge consisted of about one ton of rich ore and 500 pounds of anthracite or coke ground to pass a 16-mesh sieve and mixed with a little lime and enough water to make it plastic. This was spread as a layer about 4 inches thick on the coke- or anthracite-lined floor of the furnace and the ore reduced at a low heat. The object in using lime-washed coke in place of charcoal was to retard the combustion of the carbon and so give time for the oxygen of the ore to combine with

[13] *Journ. Iron and Steel Institute*, 1873, i, p. 37; 1877, ii, p. 345. *Trans. Amer. Inst. Min. Engrs.*, viii, p. 321.

[14] *Trans. Amer. Inst. Min. Engrs.*, 1884, xii, p. 522. "Adams's Process," *Journ. Iron and Steel Inst.* (Amer. vol.), p. 317. H. M. Howe, "Metallurgy of Steel, p. 270; I. Sterry-Hunt, "Geological Survey of Canada," Rept. Progress, 1886–9, p. 274.

[15] *Journ. Iron and Steel Inst.*, 1888, vol. ii, p. 252; 1889, vol. ii, p. 423. *Trans. Amer. Inst. Min. Engrs.*, vol. xvii, pp. 668–679.

the coke. The spongy iron obtained was used in the open-hearth and crucible processes.

Chenot's process (1851)[16] consisted in reducing the ore in externally heated retorts by either charcoal or carbon monoxide from a gas producer. The sponge was worked into a bloom in a charcoal hearth or melted in crucibles.

In Blair's process,[17] a modification of Chenot's introduced in 1870, very hot producer gas was passed through a mixture of ore and charcoal in retorts.

The chief weakness of both the Blair and Chenot process was the difficulty of economically heating the retorts and the smallness of the output.

A recent process and one which recalls the work of Siemens is the Jones [18] process, experimental work on which has been done at Iron Mountain, Mich.

These various direct processes, although industrial failures, have demonstrated that it is possible to make iron of considerable chemical purity from phosphoric ore, and, when pure magnetites are used, an iron of exceptional quality, suitable for tool steel and similar purposes.

The Application of the Electric Furnace to the Direct Reduction of Iron Ores.

In the second method of direct reduction which we have discussed,—namely, the mixing of the ore with the reducing agent and then heating it,—two prerequisites have to be met,—namely, to maintain a sufficiently high temperature to effect reduction, and to maintain a reducing atmosphere. As just stated, none of the processes devised have been able to do this in a commercially successful manner. In the electric furnace, however, it is not difficult to meet either of these prerequisites. If, for example, we place a mixture of ore and carbon in the crucible of an electric furnace of the Siemens or Heroult type, heat is generated by the resistance which is offered to the passage of the current

[16] Percy Iron and Steel, pp. 335–345; Brit. Pat. 1590, A.D. 1856.

[17] *Journ. Iron and Steel Inst.*, 1878, vol. i, p. 47; *Iron Age*, 42, p. 119; U. S. Pat. 126,922, May 21, 1872.

[18] John T. Jones, Patents: 1908—880,799, 890,229, 890,230, 890,231, 890,232, 890,233, 890,234, 890,235, 891,549, 891,704, 891,705, 899,405. 1909—12,928. 1911—981,280.

through the charge, and the heat may be increased or decreased by the amount of current passed. As it is not necessary to admit air to the furnace, the atmosphere may be kept strictly reducing. If the carbon be carefully controlled,—that is, enough carbon is provided for the reduction of the iron of the ore, but not enough to reduce other metals that may be present or to combine with the reduced iron,—a malleable iron or steel may be produced. Although many experimenters [19] have worked on this subject, and although the field appears to be a promising one, it yet remains for some one to demonstrate the feasibility of such a process in a commercial scale.

Problems in Non-Ferrous Metallurgy.

Turning our attention to non-ferrous metallurgy, let us briefly consider one or two of the problems that are now confronting the workers in that line of metallurgical work.

Concentration Problems.

As is well known, in the metallurgical treatment of many ores it is first necessary to give them a preliminary concentration treatment in order to raise the metallic content of the ore to a sufficiently high percentage to permit of its being successfully treated commercially. In other words, the object is to separate the valuable part of the ore from the worthless gangue material. In doing this it is often difficult, due to the nature of the ore, to obtain a clean separation. This is especially true in the treatment of ores when the specific gravity of the gangue material and of the mineral to be saved closely approach each other,

[19] Experiments of Stassand, "Electrochemical and Metallurgical Industry," vol. vi (1808), p. 315; *Ibid.* (1906), pp. 125, 152, 265, 332. "Metallurgical and Chemical Engineering," vol. ix (1811), p. 642.

Investigations of J. W. Evans, *Journal of the Canadian Mining Institute*, vol. ix, p. 128; *Ibid.*, vol. xiii, p. 151.

Experiments of "La Neo Metallurgie," *Revue de Metallurgie*, Dec., 1910, p. 1190.

"The Application of the Lash Process to the Electric Furnace," *Trans. American Electrochemical Society*, vol. xiv (1908), p. 239; vol. xv, p. 149.

Research work of R. M. Keeney on "The Production of Steels and Ferro-Alloys Directly from Ore in the Electric Furnace," Carnegie Scholarship Memoirs, Iron and Steel Institute (England), vol. iv (1912), p. 108.

for the majority of concentration methods are based upon the separation of minerals, generally in water, by reason of the difference in their specific gravities. Difficulties are generally encountered when attempting to treat ores which have to be very finely ground in order to free the metallic minerals from the gangue minerals, or when a portion of the metallic minerals may be in the form of carbonates. Due to causes of this kind, it may not be possible to save by concentration more than 55 to 65 per cent. of the metallic mineral contents of the ore. Thus we see that if true progress in metallurgical practice means conservation, there is a big field for improvement in the treatment of those ores which of necessity must be concentrated before they can be economically treated by present-day metallurgical processes. In other words, the problem resolves itself into devising more efficient methods of concentration, or else of devising processes which can economically treat the ore without its having to be concentrated.

The Treatment of Complex Ores.

As was only natural, in the winning of metals from their ores, especially in this country, those ores were treated which would most easily give up their metals. Take, for example, gold: It was first obtained from the river beds by panning, by cradling, and by sluicing. The river deposits becoming worked out, the miners began to prospect. They discovered the gold-bearing minerals in place. The gold was in the native state, and all they had to do was to crush the ore and amalgamate the gold. Gradually the nature of the ore changed, and soon parts of it contained gold values which could not be extracted by amalgamation. This meant that concentration had to be resorted to in order to save these valuable gold-bearing particles from the tailings of the stamp battery. Then processes had to be devised for treating the concentrates. As time went on it became necessary for metallurgists to turn their attention to the treatment of ores more and more refractory and complex, until at the present time the metallurgical profession is confronted with the problem of successfully treating such ores. In a complex ore the metallic minerals (sulphides) which generally predominate are those of zinc, iron, copper, and lead, and it is generally zinc which gives the trouble. As pointed out by Dr. Parsons in Bul-

letin No. 47 of the Bureau of Mines, "besides the losses in mining and concentration of zinc ores, there are incalculable losses, which, without question, run into millions of dollars, and undoubtedly exceed the total value of the zinc mined, in slags and waste products, from other processes. Zinc has been and generally still is considered about the worst impurity to be found in the ores of copper and lead, for it has always given trouble in their metallurgy. Accordingly, the practice has been so to run the charge that the zinc passes off in the slag, and to a certain extent in the flue dust The economical treatment of some ores has been impossible on account of the high percentages of zinc they contained. . . . To make even an approximate estimate of the total loss of zinc is impossible, but it certainly exceeds thousands of tons daily."

Much attention is being given to this subject. Perhaps none of the processes which have been proposed will be the final solution of the problem, but rather that some one process will be found to be applicable under certain conditions, while under other conditions some other process will best meet the requirements. For the sake of convenience, we may classify the methods which have been proposed under the following headings:

(1) Special smelting processes.
(2) Chemical processes.
(3) Electric-furnace processes.

In this connection, we will not attempt to do more than to briefly consider the electric furnace method of zinc smelting.

PRESENT METHODS OF ZINC SMELTING.

Briefly outlined, the process is as follows: The ore is roasted, for the purpose of converting the zinc into zinc oxide. It is then mixed with the proper amount of reducing agent and placed in a closed fire-clay retort, to which is attached a condenser. The retort is kept at a temperature of from 1100° to 1300° C. The zinc oxide is reduced to zinc at a temperature of 1033° C. As the boiling point of zinc is somewhere between 920° and 930° C., the reduced zinc is at once vaporized, and then subsequently condensed as a liquid metal in the condenser, within the temperature range of 860° C. down to 420° C., the higher temperature being that at which metallic zinc approaches the vaporization point, while the lower temperature is that of the solidifica-

tion of zinc. In order to successfully carry out the process as above outlined, the temperature and the vapor must be carefully controlled, for if the temperature gets too low, or the vapor too dilute with carbon dioxide instead of carbon monoxide, or with metallic vapors other than zinc, what is known as blue powder is formed, which consists of minute particles of zinc, each one apparently covered by a thin coating of zinc oxide. If formed, this blue powder is a source of a great deal of trouble, as it cannot be converted to spelter by a single melting.

As a retort holds only about 200 pounds of mixture, the process is necessarily an intermittent one. After a charge has been smelted, the zinc is removed from the condenser, the residue is raked out of the retort, and the latter is then ready for another charge. Due to the process being an intermittent one, the labor cost is comparatively high, and, as the heat has to be conducted to the charge through the walls of the retort, the thermal efficiency of the process is low.

The Use of the Electric Furnace in Zinc Smelting.

Due to the drawbacks of the present method of zinc smelting, as above mentioned, various attempts have been made to obviate them by the use of the electric furnace. In fact, more progress has been made to date in the electric smelting of zinc ores than with the electric smelting of any of the non-ferrous metals, except aluminum and metals which are used in the production of ferro-alloys. The processes which are used in the electric smelting of zinc ores are of two general classes. The first of them is based upon the reduction of zinc oxide and its compounds by carbon and carbon monoxide; the second, upon the decomposition of zinc sulphide by metallic iron.

Reduction by Carbon and Carbon Monoxide.

Considering, now, the first of these processes,—namely, the reduction of zinc oxide and its compounds by carbon and carbon monoxide,—the one essential difference between reduction in a retort and in an electric furnace in this manner is that in a retort the process is intermittent, whereas in the electric process it may be continuous. If continuous, the furnace is charged at intervals, as may be necessary, and the metal is recovered from the condenser just as in the retort process.

On the other hand, as above stated, the slag in the retort process is raked out of the retort, as it is never fluid enough to be tapped out, whereas in the electric furnace it is tapped out at intervals, or is allowed to run from the furnace continuously. In either case the continuous operation of the furnace is not disturbed.

Although in an electric-furnace process of this kind the reduction of the zinc oxide by carbon or carbon monoxide occurs just as in the retort process, the reduction seems to take place much more rapidly in the electric furnace, as designed at the present time, and the reaction $CO_2 + C = 2CO$ does not seem to take place as well in the electric furnace as in the retort. On this account the atmosphere of the electric furnace contains a larger per cent. of carbon dioxide than does the atmosphere of the retort, and as a result a larger amount of blue powder is formed in the electric furnace during the condensation of the vapors than is formed in the retort.

The Use of Iron as a Desulphurizing Agent in Electric Zinc Smelting.

There are at least two electric furnace processes[20] for the smelting of zinc ores which are based upon the use of iron as a desulphurizing agent. However, they do not seem to have advanced as far as reduction of the oxide with carbon. The same trouble is experienced in the use of iron as a desulphurization agent as when carbon is used as a reducing agent,—namely, in condensation a large amount of blue powder is formed which has to be resmelted. This is probably due to the volatilization of iron and iron sulphide, which, together with the other vapors

[20] These processes are the Imbert and Cote-Pierron processes. "Imbert tried out the reaction without much success in the combustion furnace. Thompson and FitzGerald later designed an electric furnace for its application. Although the Imbert-Thompson-FitzGerald process has been tried experimentally on apparently a good-sized scale for two years in Upper Silesia, little authoritative information has been given out concerning it. Several types of resistance furnace have been used, all having a resister of solid carbon."

"The Cote-Pierron process is performed in a furnace of the Siemens type, or one having two vertical electrodes in series, both types being roofed and connected with a column of electrically-heated carbon, through which the vapor passes before going to the condensing chamber."

that are given off, dilute the zinc vapor and so bring about the formation of blue powder in condensation, as is the case when zinc oxide is reduced with carbon.

The Problem to be Solved in Connection with the Electric Furnace Smelting of Zinc Ores.

From what has just been stated, it is evident that the problem in connection with the use of the electric furnace in the smelting of zinc ores lies in devising a furnace which may be operated in such a manner as will prevent the formation of blue powder instead of metal, under the peculiar conditions of the electric furnace. The problem is, therefore, not an electrical or mechanical problem, but a metallurgical problem, and its solution, as before stated, lies in devising a furnace in which the metallurgical difficulty now met with may be successfully overcome,—namely, the presence in the vapors of the furnace of an excessive amount of carbon dioxide, with the consequent formation of blue powder.

It must not be inferred from what has been stated that the electric furnace is not being used commercially for the smelting of zinc ores. As a matter of fact, it is, but, so far as the writer is aware, only in Sweden and Norway, where some 8000 to 10,000 electric horsepower is used for that purpose. Not much has been published as regards the work which has been done in these countries, except a report by F. W. Harlord,[21] published in 1911, in which he states that the weak point of the process is the large amount of metallic powder produced in proportion to the metals, as two tons of blue powder were being smelted at Trollhättan, Sweden, for each ton of ore treated.

THE SMELTER FUME PROBLEM.

The ores which are smelted in non-ferrous metallurgical work are largely of the sulphide class. Such ores may either be given a preliminary roast and then smelted, or they may be smelted directly in the blast furnace. In either case obnoxious fumes are given off,—that is, certain portions of the charge may be volatilized either in the form of metals, oxides, sulphides, sulphates, or chlorides of metal. Sulphur is converted for the most part into

[21] *Engineering and Mining Journal*, Feb. 10, 1912, p. 344.

sulphur dioxide, and, under certain conditions favorable to the same, the sulphur dioxide is further oxidized to sulphur trioxide (SO_3). All of these products,—that is, the fine particles of ore, the volatilized products, and the sulphur gases,—pass off with the furnace gases. By means of mechanical devices, such as bag houses, or by means of the Cottrell Process,[22] the solid particles can be removed from the furnace gases, but so far no mechanical device has been designed which will handle the sulphur gases, which, if present in appreciable amounts, are very destructive of the vegetation which may be found in the region adjacent to the smelting plant. An idea of the large volume of these gases may be gained from the fact that in smelting 1000 tons of pyrite ore, approximately 800 tons of sulphur and oxygen are discharged into the atmosphere. About the only method which has proved commercially effective, so far, for the removal of sulphur trioxide from furnace gases has been the absorption by water, and this is not commercially feasible, unless it is desired to manufacture sulphuric acid; and, again, it is not commercially feasible to manufacture sulphuric acid unless the smelting works are located in some section of the country from which the acid may be easily marketed. In other words, although it may be feasible to convert sulphur gases into sulphuric acid at Ducktown,[23] Tenn., it is not feasible to attempt to do so at Great Falls or Anaconda, Mont., as there is a comparatively limited demand for sulphuric acid in that part of the country at the present time. The problem, therefore, for such districts as Great Falls and Anaconda resolves itself into one of recovering the sulphur from the smelting operation in the form of elemental sulphur or as some by-product, as is done by the Sprague[24] process, in which process the sulphuric acid is neutralized by zinc oxide, lime, or magnesia, and the fume removed by filtration through

[22] F. G. Cottrell, "Electrical Fume Precipitation," *Trans. Amer. Inst. Ming. Engrs.*, Feb., 1912.

Linn, Bradley, "The Electrical Precipitation of Suspended Particles by the Cottrell Process," *Journal of Industrial and Engineering Chemistry*, vol. iv, No. 12, Dec., 1912.

[23] Freeland, W. H., and Renwick, C. W., "Smelting Smoke as a Source of Sulphurous Acid," *Eng. and Min. Journ.*, May 28, 1910, p. 1116.

[24] C. B. Sprague, "Sprague Process for Treating Furnace Gases," *Eng. and Ming. Journ.*, Mar. 5, 1910, p. 519.

fabrics. A great deal of thought has been given to this subject, and a great deal of experimenting done. At the present time there are two processes which are now being watched by the non-ferrous metallurgical world with interest. The first of these is the Thiogen [25] process, which is based upon the fact that by uniting sulphur fume with carbon and then passing the product over limestone containing iron salts the gas is decomposed and sulphur precipitated. In the Thiogen system an oil spray is used to supply the carbon, while calcium sulphide is used as an accelerating agent.

The second and most recent process is that of Hall, who proposes to remove the sulphur from the ore direct by distillation, and to thus prevent the formation of oxidized sulphur gases which must be subsequently reduced or neutralized. In order to obtain the sulphur in the manner above indicated, the ore is roasted at a temperature between 700° and 900° C. in a neutral or non-reducing atmosphere, a certain amount of water as steam is introduced into the furnace during the roasting operation. The base metals are thus oxidized and the sulphur is distilled without becoming oxidized, and is collected in a gas washer or by some other suitable means.

HYDRO-METALLURGY OF COPPER.

In closing, perhaps I can do no better than to cite the hydro-metallurgy of copper as one of the great problems which now confront the non-ferrous metallurgist. As stated in the *Engineering and Mining Journal*, in a recent editorial on this subject, it is perhaps the greatest. While it is only recently that this subject has attracted very much attention in this country, it is by no means a recent innovation, as hydro-metallurgical methods of copper extraction have been used in Portugal and Spain for a long time, being particularly suited to the peculiar conditions to be found there, but they have never been employed to any large extent in this country. In general the process consists of getting the copper values into a soluble form, if they are not already in that condition. Sulphuric acid is generally used as a leaching agent. The copper may then be precipitated out of solution by

[25] "The Thiogen System of Fume Control," *Metallurgical and Chem. Engineering*, vol. x, 1912, p. 710.

any one of three methods,—namely, by iron, by hydrogen sulphide, or electrolytically. There is still a difference of opinion as to which of these methods is preferable. There is still much work to be done on this problem, but it is not believed that the difficulties are insurmountable, and, moreover, it is confidently believed that the process will be developed along modern lines and in accordance with modern practice, until the hydro-metallurgy of copper shall have reached a stage of development comparable to that of the hydro-metallurgy of gold and silver ores, as exemplified in the cyanide process. In other words, it is confidently predicted by those who are familiar with the work that we are to have a new metallurgy of copper. As can be readily understood, the hydro-metallurgy of copper is one solution of the fume problem, mention of which has previously been made.

There are doubtless many other metallurgical problems to be solved, mention of which could be made by those present, but it is believed that those named will suffice to give some idea of the nature and extent of the problems with which the metallurgist is confronted at the present time.

In closing, the writer wishes to express his thanks to his colleagues, Messrs. R. M. Keeney and J. F. Cullen, for data furnished in preparing this paper.

PITTSBURGH, PA., December, 1913.

Société Francaise de Photographie. The French Society of Photography at Paris announces a photographic exhibition to be held in the spring of 1914. The exhibition is open to all photographers, amateur and professional, members and non-members of the society. Material for exhibits should be sent to 51, Rue de Clichy, Paris (9e), France, before the 15th of April, 1914. The exhibitors must present the results of their own labor only and subjects which have not formerly been exhibited. The exhibition will be divided into five sections:

1. Prints intended to be framed under glass. 2. Lantern slides. 3. Stereogrammes of all kinds. 4. Scientific subjects. 5. Reproductions by various photographic processes in black and in colors.

Exhibitors may show only a limited number of pictures in each section. Medals of silver and bronze will be awarded. A special medal in each class will be reserved for work executed in the studio of the society.

Further information may be obtained by addressing the society as above.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

MICROMETER MICROSCOPES.

AFTER pointing out numerous sources of error generally overlooked when attempting to make accurate length measurements with micrometer microscopes, the author describes a simple and rapid method of applying the proper corrections. He then explains an accurate means of measuring screw errors and a graphical procedure for computing convenient correction tables.

The advisability of carefully examining any instrument intended for accurate work is made clear by a table giving the magnitudes of the periodic errors found in some microscopes. In this connection it is pointed out that, even with such micrometers as those of the International Bureau of Weights and Measures, an error of about 1.5 can be made in comparing two lengths if the only corrections applied are those for deviations of the mean screw-values from the nominal value of 100 per revolution; and, further, that most of the error will be due to neglecting the *periodic* corrections.

After a pair of comparator microscopes have been calibrated once for all, the necessary corrections may be fully taken into account with an expenditure of labor even less than that required in the common but inaccurate practice of multiplying the changes in each micrometer reading by the mean scale-value and subtracting. The method of doing this is essentially a simplification of the procedure followed by the International Bureau in measuring the national prototype meters. The principal gain is effected by reducing to two tables (both of which can be mounted on a single card) all the information contained in the six tables used by the Bureau. Another gain is effected by recording observations in such a form that the corrections can be added directly without any copying of figures. The advantages of the method are illustrated by examples of actual measurements.

The calibration of a microscope by one of the processes usu-

* Communicated by the Bureau.

ally described involves such laborious calculations and least-square adjustments that it is no wonder to find it attempted only as a last resort when the demands of high accuracy compel. While, of course, the complete calibration of any divided scale requires considerable time, it is, however, possible to determine microscope corrections more accurately and with less labor than would appear from previous accounts. The whole procedure is made one of direct measurement and simple graphical addition, which is illustrated by a concrete example. The displacements of the spider-lines that correspond to certain calibration intervals are measured with an auxiliary micrometer microscope. In this way far greater accuracy is secured than is possible in the customary method of using the ocular of the micrometer under investigation. From these measurements are plotted curves which show how the intervals vary in length throughout the course of the screw. After these are adjusted so as to be mutually consistent, they are combined to form a new series from which the corrections due to screw irregularities may be read off immediately. The magnification of the microscope is then adjusted, and determined by measuring a known length. This fixes the slope of a straight base line which corrects for deviations of the nominal from the actual scale-value. A translating scale is now used for the final step of reading off the corrections to be entered in the table.

ABSTRACT OF BUREAU OF STANDARDS TECHNOLOGIC PAPER ENTITLED,

"SOME LEADLESS FRITTED GLAZES."

By E. T. Montgomery,
Assistant Ceramic Chemist.

IN the manufacture of American whiteware, china, and porcelain, and of English bone-china, a glaze composed of borosilicates of the alkalies, lime, lead, and alumina maturing at a relatively low temperature (about Seger cone 3, approximately 1100° C.) is applied to a body previously fired to the desired maturing point for the given body, approaching vitrification more or less closely. It is essential in this process that the maturing point of the glaze be considerably below that of the vitrification temperature of the body in order to avoid, in the glost fire, undue softening of the body and consequent deformation and loss. To

accomplish this purpose and at the same time to secure their inherent qualities of long temperature range, brilliancy and low viscosity, such low fusing glazes as borosilicates containing lead oxide are employed.

Chiefly on account of the poisonous nature of the lead compounds used in making these glazes, the subject of leadless glazes has aroused the interest and has received the serious attention of both technical ceramists and of manufacturers for many years. The present work was undertaken for the purpose of investigating the merits of the various types of leadless glazes thus far reported, of studying their properties and possibilities, of improving them if possible, and of making a general comparison, from a technical standpoint, between lead and leadless glazes for whiteware and china at a glost heat of Seger cone 3 (approximately 1100° C.).

To this end numerous series of glazes were made up, being so planned as to include a number of previously-reported leadless glazes of merit, and all methodically arranged in such a way as to fully cover the range of composition which it was believed desirable to study. All of the glazes were tested over six standard underglaze colors of known composition, and all of their properties were studied in direct comparison with a standard whiteware or china glaze made with lead, half of each trial being dipped in this standard lead glaze and half in the leadless test glaze.

The results of this investigation developed many interesting technical points both in regard to leadless glazes in themselves and in comparison with lead glazes. In this brief abstract space will not admit of giving these detailed results. In general, we may say that, while lacking some of the valuable characteristics and qualities of lead glazes, leadless glazes have a few distinct advantages of their own. On the other hand, it is the writer's opinion that it would be premature at the present time to say that we can make a leadless glaze for whiteware, china, or porcelain which could be substituted for the present lead glaze. Both have special faults and special virtues, but for use under general factory conditions, make it as perfect as we can, we are not likely to find any element, compound, or combination of compounds which will exactly duplicate the many excellent properties of lead as a glaze ingredient.

THE PENTANE LAMP AS A WORKING STANDARD.

By E. C. Crittenden and A. H. Taylor.

[To be published in the *Bulletin of the Bureau of Standards* and in the *Transactions of the Illuminating Engineering Society*.]

THE best standards of candle-power are electric incandescent lamps, but it is not always practicable to use them. Of the various flame standards, the Harcourt 10-candle pentane lamp appears to be the best for general use. This lamp has the disadvantages of being large and not easily portable, of using fuel which is expensive and somewhat dangerous, and of requiring more air than ordinary ventilation will supply, but in spite of these faults its use is increasing rather rapidly.

The candle-power is not usually exactly ten, and the exact value must be determined by a photometric test. The intensity is affected by atmospheric conditions, but when the proper corrections are made a given lamp will repeat its value very closely.

It is found that the pentane commercially obtainable increases in density rather rapidly by the fractional distillation which occurs in the lamp. The density (at 15° C.) should be between 0.6235 and 0.626, but it usually reaches 0.635 when a little over half of the liquid has been used. Beyond this point there is a marked increase in the intensity of the flame. For an approximate correction, the variation may be assumed to be linear and to be one per cent. in candle-power for each 0.01 in the density of the liquid.

When the lamps are operated in a well-ventilated room, the most important cause of variation is the humidity. Water vapor in the air lowers the intensity of the flame, and variations due to that cause may exceed 15 per cent. Previous work at the Bureau of Standards * has indicated that one per cent. of water vapor in the air causes a decrease of 5.67 per cent. in the candle-power of the flame, whereas the correction officially established in England is 6.6 per cent. The present paper gives further data based on about 75,000 individual photometer settings on 27 lamps, including all lamps tested in 1911 and 1912 whose tests have extended over a range of 0.5 per cent. or more of water

* Rosa and Crittenden, *Transactions Illuminating Engineering Society*, 5, 1910, p. 753.

vapor in the air. The mean result is an exact check of the correction factor previously found at the Bureau.

To facilitate the reduction of observations to normal candlepower, a chart is given from which the departure of a lamp from normal value can be read directly when the barometric pressure and the readings of the wet and dry bulb thermometers of a ventilated hygrometer are known. The chart is plotted for a pentane lamp, but it may be applied to other flames without introducing serious errors.

TECHNOLOGIC PAPER NO. 15.

SURFACE INSULATION OF PIPES AS A MEANS OF PREVENTING DAMAGE BY ELECTROLYSIS.

By Burton McCollum and O. S. Peters.

AN investigation of the subject of surface insulation of pipes as a means of preventing damage to underground metallic structures by stray currents from electric railways has recently been completed at the Bureau of Standards. Tests were made of the various substances available for the purpose of insulation of underground structures, including paints, pitch and asphalt dips, pitch and paper and asphalt and felt wrappings, and so forth. Test specimens were made by lining shallow sheet-iron cones with the material to be tested. Before being subjected to the final test each cone was filled nearly full of ten per cent. salt (NaCl) solution and an alternating difference of potential of 80 volts (effective) applied across the coating for 30 seconds, in order to be sure that it was continuous and without flaws. A milliammeter in series with the specimen indicated a defective coating by a kick of the needle. The electrical resistances of the perfect specimens were then approximately determined with a Wheatstone bridge. In the case of the paints these resistances were found to be of the orders of from 10^5 to 10^{11} ohms per square centimetre, while for the wrappings they were much higher.

The final test of the specimens which survived the preliminary test consisted in allowing water and air alternately to come in contact with the coating while a direct potential difference of either four or fifteen volts was applied across the coating. The

value of the voltage applied depended on the thickness and other characteristics of the coating. In some of the specimens made up from each material the iron of the cone was made negative and in others positive, while in the case of the paints some of the specimens were subjected to the alternate action of air and water with no potential difference applied, in order to check up the effect of the electric stress.

The alternating contact with the coating of air and water was obtained by filling the cone and allowing the water to evaporate, which took about a week. Readings of the current flow were made at suitable intervals. The first appearance of current flow was taken as indicating the end of the useful life of the specimen as an insulating coating.

The average life of the paints was about 116 days, the maximum life obtained from any specimen being but little more than a year. No conclusive evidence was obtained that the low potential differences applied had any effect in hastening the initial failure of the coatings. The wrappings lasted longer than the paints and dips, but none of them much more than 400 days. It seems from the results that the failure of the coatings must be caused by absorption by them of water, which in time penetrates to the iron, allowing current to flow and destroy the coating by electrolysis. After the first appearance of current flow the destruction of the coating was observed to proceed very rapidly.

The conclusion drawn from the results of the laboratory tests,—*i.e.*, that the protection against electrolysis which is obtained by wrapping or painting pipes or other metallic bodies for use underground is only temporary,—is borne out by tests on insulated pipes buried in the ground under practical conditions, and also by correspondence with gas and water companies whose experiences lead to the same conclusion.

Protective Rubber Coating for the Hulls of Ships. R. DITMAR. (*Z. Angew. Chem.*, xxvi, 565.)—It is stated that coatings composed of rubber mixings, raw or semi-vulcanized, prepared from low-grade rubbers rich in resin, and preferably containing large proportions of factis (a rubber substitute) and inorganic fillers, when pierced by a projectile will close over the hole and so prevent the inrush of water. The coating may be made more resistant to water by adding tar. It is laid on or cemented to the hull of a ship and painted to protect it from attack by marine animals.

THE FRANKLIN INSTITUTE

(Proceedings of the Annual Meeting held Wednesday, January 21, 1914.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, January 21, 1914.

PRESIDENT WALTON CLARK *in the Chair.*

Additions to membership since last report, 12.

The tellers, Messrs. Colvin, Jennings, and Williams, submitted the report of the ballots cast for President, Vice-President, Treasurer, and members of the Board of Managers, and the following gentlemen were declared duly elected to the respective offices:

Walton Clark, President (to serve one year).

Coleman Sellers, Jr., Vice-President (to serve three years).

Cyrus Borgner, Treasurer (to serve one year).

John Birkinbine, Walton Forstall, E. Goldsmith, Harry F. Keller, Louis E. Levy, Richard Waln Meirs, Alexander P. Robinson, George D. Rosengarten, Managers (to serve three years), and Marshall S. Morgan, Manager (to serve two years).

The following proposed amendment to the By-laws, presented at the December meeting, was submitted for final action and adopted:

Article III, Section 10 of the By-laws to be amended as follows:

After "Philadelphia" in the second line add: "except that members of the Army and the Navy shall be considered as non-resident members, irrespective of residence," so that the section, as amended, shall read:

"Non-resident members shall be those who reside permanently at a distance not less than twenty-five miles from Philadelphia, *except that members of the Army and the Navy shall be considered as non-resident members, irrespective of residence.* Transfers of membership from the resident to non-resident class may be granted by the Board of Managers at its discretion in cases of temporary absence of a member from the city for a period of not less than one year."

The President then presented a statement of the work of the Institute during the past year, and submitted the reports of the various standing committees.

The paper of the evening, on "The Heat Received by the Earth from Stars and Planets," was presented by Dr. Ernest F. Nichols, President of Dartmouth College, Hanover, N. H. The speaker gave an interesting account of the experiments carried on at the Yerkes Observatory in the summer months of 1898 and 1900, directed toward detecting and, if possible, measuring the heat received from the two stars Arcturus and Vega and the planets

Jupiter and Saturn. The apparatus and experimental methods used, as well as the numerous obstacles which were encountered in the conduct of the work, were fully described. The numerous data recorded during the course of experiments were presented. The subject was illustrated by lantern slides. In the brief discussion which followed the presentation of the paper, Messrs. Levy, Henderson, and Bond participated.

A rising vote of thanks was extended to the speaker.

Adjourned.

R. B. OWENS,
Secretary.

REPORT OF THE BOARD OF MANAGERS FOR THE YEAR ENDING SEPTEMBER 30, 1913, WITH APPENDICES EMBRACING THE ANNUAL REPORTS OF THE VARIOUS COMMITTEES AND SECTIONS.

To the Members of The Franklin Institute:

Your Board of Managers formally present herewith the reports of the standing committees upon the operations of the past year. These reports have been printed, and a copy will be sent to each member of the Institute. The Board hope that you will come from the reading of these reports satisfied that the work of the Institute has been conducted with industry and intelligence by those to whom you have entrusted it.

The Committee on Instruction, Mr. Paul, Chairman, report an increase in the enrolment of students and in the number of graduates for this, the ninetieth consecutive year of the school's activity. The Institute has recently greatly improved its school facilities, and is rewarded therefor by a most gratifying increase in the student roll. The registration of students for the winter term has steadily increased from 173, in 1909, to 306, in 1913, and the number of graduates has increased in practically the same proportion. We have now as many students on our rolls as we can properly care for in the space that we are able to devote to this purpose. Our effort of late has not been to increase the attendance at our schools, but to improve the quality of the service that we render to the individuals and to the community through that branch of our work.

The report of the Committee on Elections and Resignations of Members, Mr. Eglin, Chairman, shows a gratifying increase in the membership—now numbering 1395. We have elected, during the past year, 292 members, which is the largest enrolment for any year since the present rate of dues was established. We have increased nearly 50 per cent. in membership in the past five years. The record of resignation of members shows that the resignations are much less, in proportion to the total enrolment, than we had, unfortunately, become accustomed to expect. This is one of the most convincing and gratifying evidences of an increased interest in the work the Institute is doing for the public good, as well as an increased appreciation of what the Institute does for the individual member.

We learn from the report of the Library Committee, Mr. Levy, Chairman, that there have been 3306 additions to the library during the past year, which is about 10 per cent. more than in the previous year, and 50 per cent. more than the increases in 1909. The library is used more and more largely from year to year. The pressing demands of the library to-day are for increased space and more convenient location.

The report of the Committee on Publications, Mr. Levy, Chairman, while interesting, calls for no special comment. I desire, however, to again call your attention to the JOURNAL. Within the past few years there has been created a board of associate editors, comprising twenty men, of international reputation for scientific work. This is an added, and the most certain attainable, assurance of value and accuracy in our senior publication. The size of the JOURNAL, during the last five years, has been increased about 25 per cent. Its standing in the scientific world, always notable, has been fully maintained.

The Year Book of the Institute, brought out by the Committee on Publications, is a worthy publication, setting out the activities of the Institute, and containing historical notes, the programme of lectures, the charter and by-laws, and a list of the membership.

The Committee on Meetings, Mr. Rogers, Chairman, report monthly meetings from October to May, inclusive, at many of which the capacity of the Hall was overtaxed. The subjects treated covered a wide field of scientific and engineering activity, as will be seen by reference to the JOURNAL for the past year.

The Committee on Sectional Arrangements, Mr. Day, Chairman, report an even better attendance during the past year than in previous years. The committee report sixteen meetings during the year, each with a lecture from some man of acknowledged authority in his line of work. The high standard for these meetings, set for us by our predecessors, is being fully maintained by the members of the Sections and of your Committee on Sectional Arrangements.

The report of the Committee on Exhibitions, Mr. Birkinbine, Chairman, contains no present recommendation, but suggests the possibility of an exhibition in the near future.

The Committee on Museums, Professor Outerbridge, Chairman, presents an interesting report, covering, in the main, a list of gifts received during the year and placed in charge of this committee. Some of these gifts are of great historic interest. The Board respectfully suggest that you, members of the Institute, do not sufficiently appreciate the number and interest of the articles in charge of the Committee on Museums. They are now all available for convenient inspection.

The report of the Committee on Science and the Arts, Mr. Crisfield, Chairman, shows great activity. It contains certain recommendations that will be considered by the Board and by the committee for the year 1914. The committee report the disposition of twenty-seven cases during the year, with seventeen awards made. The report shows the very broad field covered by the committee's efforts; and in the list of awards made will be found the names of men among the most distinguished in their respective branches of scientific work.

The report of the Finance Committee, Mr. Forstall, Chairman, shows that for the year covered by the report the Institute had an income applicable to operating of..... $29,887.73
that the operating costs were 34,158.42

making a deficit for the year's operation of $4,270.69

Our financial condition has improved of late years. We have some reason to believe that our days of deficits will soon be past.

In this connection we make the following memorandum of endowments received during the past three years:

John H. Noblit	$135,922.79
James H. Cresson	50,000.00
Maria Blanchard	10,000.00
Anna Blanchard	10,000.00
Mary Thorn	1,000.00
Charles Longstreth	1,000.00

Through the generosity of Mr. Samuel Insull, of Chicago, we are able to provide an additional and very valuable gold medal, to be known as the Franklin Medal, and to be awarded for general scientific achievement, leaving the Elliott Cresson Gold Medal to be awarded for specific achievement in science. Your Board think this medal makes complete our list of awards, enabling The Franklin Institute to satisfactorily evidence its approval of notable work for the good of mankind in any part of the broad field of science.

Since our last report to you the widow of our good and lamented friend and one-time secretary, Dr. Wm. H. Wahl, has laid down the burden of her mortality. This fact is worthy of record, both because of the distinguished services that Dr. Wahl rendered to the Institute and because of the fact that, under his will, his estate was to descend to the Institute, upon the death of his wife—conditioned on the Institute's raising a sum equal to the value of the estate. The present value of the estate is about $85,000. It is incumbent upon the membership to secure that this munificence be not lost to us. This award being secured, there will be no further anxiety as to the financial support of the work the Institute is doing for the good of the citizen and of the State.

Your Board have satisfaction in reporting to you the purchase by your Franklin Fund and Building Committee, Mr. Howson, Chairman, of 23,320 square feet of land and the eight residences thereon, fronting Logan Square and the route of the boulevard. The land lies on the southeast corner of Race and Nineteenth streets. The Parkway frontage is 110 feet. Among our near neighbors, when we have secured the necessary further funds and erected a building upon this incomparable site, will be the Cathedral, Wills Eye and Ear Hospital, the Academy of Natural Sciences, and the great and noble new Public Library.

Your Board regard the acquisition of this valuable and improving real estate by the Institute as a notable achievement for the cause of the

scientific advancement of the mechanic arts in the State of Pennsylvania, and congratulate you upon it.

While we look with hope to the day when a new building, larger than this in which we now gather and on the more convenient site, will enable us to better serve the citizen and the State, we will not abate our efforts in the interim to carry on the work of The Franklin Institute in this beautiful and historic building in a manner as nearly may be worthy of the confidence you have shown in us, and of the standards set by the able and devoted men who have preceded us.

WALTON CLARK,
President.

REPORT OF THE COMMITTEE ON INSTRUCTION.

To the Board of Managers:

The Committee on Instruction has been much gratified by the favorable showing made by The Franklin Institute School of Mechanic Arts during the past school year and by the marked increased interest shown by the members of the Institute in the course of Popular Lectures.

The School, now in the 90th year of its existence, opened its winter course in September, 1913, with an enrolment of 306 students, an increase of 20 over the corresponding term of the preceding year. The plan of instruction was similar to that of the previous year, the work of the School being divided into four departments, Drawing, Mathematics, Mechanics and Naval Architecture.

The faculty for the year consisted of:

Professor William H. Thorne, Director and in charge of the Department of Drawing.

Professor William E. Bullock, Assistant Director and in charge of the Department of Mechanics.

Professor H. P. Tyson, in charge of the Department of Mathematics.

Professor H. C. Towle, in charge of the Department of Naval Architecture.

Professor Thorne was ably assisted in his department by Professor Clement Remington, in charge of Architectural and Freehand Drawing, and by Messrs. F. H. Lobb, I. P. Pedrick, C. Rommel and W. W. Twining, Assistants in Mechanical Drawing.

Professor Bullock had in Messrs. Bark and Wheatley very competent assistants.

Throughout the year the classes were well attended and the standard of work was exceptionally good. Seven visits were made on Saturday afternoons during the season to some of the most prominent industrial plants, and also special invitations were given to the students to attend such of the Institute lectures as were of direct interest to them.

The visits made were as follows:

The Ogontz Power Plant of the Phila. Rapid Transit Co.,

The Belmont Pumping Station,

The Power Plant of The Baldwin Locomotive Works,
The University of Pennsylvania Engineering Laboratories,
The United Gas Improvement Co., Station B Works,
The U. S. Navy Yard, League Island,
The Philadelphia Electric Co., 26th and Christian Streets.

The greatest courtesy was shown to the party at each plant visited, and the thanks of the Committee are extended to the management and employees of these establishments for the invitations extended and for the attention bestowed.

The graduating exercises were held in the Hall of the Institute, on the evening of April 18, 1913. Mr. Walton Clark, President of the Institute, occupied the chair, and the graduating address was delivered by Mr. G. R. Henderson, Consulting Engineer of The Baldwin Locomotive Works. Forty-five students were graduated, twenty-six in the Department of Drawing, nine in Mathematics, seven in Mechanics, and three in Naval Architecture; one student was awarded the New York Shipbuilding Company's prize for the student of greatest merit in Naval Architecture; fourteen students in the Department of Drawing received Scholarships from the B. H. Bartol Fund, entitling them to free tuition; forty students were awarded Honorable Mention Certificates for the Winter Term, and thirty-four for the Spring Term. The announcement was made at the graduating exercises that Mr. J. B. McCall, of the Philadelphia Electric Company, had offered a prize for the student of greatest merit in the Department of Mathematics; Mr. Wilfred Lewis offered a similar prize in the Department of Mechanics; the New York Shipbuilding Company again presented a set of drawing instruments to be awarded to the student of greatest merit in the Department of Naval Architecture; the Alumni Association of The Franklin Institute offered special prizes for the most proficient students in Mechanical Drawing, Architectural Drawing, Freehand Drawing, Mathematics and Naval Architecture, and also free membership for one year to graduates in each Department having a 100 per cent. attendance record, or if the total number of graduates having such record is less than ten, to the ten graduates having the best attendance record. The above prizes will be awarded on the results of the present year's work.

The Committee on Instruction has pleasure in reporting that all those connected with the School have shown the utmost zeal in promoting its interest, and the successful year is due to the thoroughness of their instruction and to their earnest co-operation with the Committee.

On January 31, 1913, the first of the Popular Lectures was given at the Institute; the lecturer being Dr. G. F. Stradling, and the subject, "On the Edge of Alaska." This was followed on February 6, by Mr. W. N. Jennings, who spoke on "Camp Life in Philadelphia," and on April 26, Mr. Fred Monsen lectured on "The Trail of the Spanish Pioneer." That the lectures were thoroughly appreciated, was attested by the enthusiasm of the large audiences.

Our thanks are extended to those gentlemen who have so generously given their services in the course of popular lectures referred to above, and who, by their help, have greatly assisted the Institute in its educational work.

Summary of the School Registration as compared with the preceding year:

	Winter Term 1912–1913	Winter Term 1913–1914	Spring Term 1912–1913	Spring Term 1913–1914
Drawing	158	188	132	..
Mathematics	61	71	39	..
Mechanics	38	30	23	..
Naval Architecture	29	17	23	..
	286	306	217	..

Respectfully submitted,

LAWRENCE T. PAUL,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON ELECTIONS AND RESIGNATIONS OF MEMBERS.

To the Board of Managers:

During the fiscal year, ending September 30, 1913, there were two hundred and ninety-one members enrolled; resignations were received and accepted from thirty-one members; and deaths were recorded of nineteen members. Four resident members were transferred to the non-resident class, and one non-resident to the resident class.

The figures for the several classes of membership and for the two preceding years follow:

Elections:	1910–11.	1911–12.	1912–13.
Resident Members	28	68	101
Non-Resident Members	15	33	186
Associate Members	5	7	1
Honorary Members	0	1	0
Life Members	4	1	3
	52	110	291

Resignations:	1910–11.	1911–12.	1912–13.
Resident Members	12	27	16
Non-Resident Members	12	13	13
Associate Members	0	1	2
	24	41	31

Deaths:	1910–11.	1911–12.	1912–13.
Resident Members	16	9	8
Non-Resident Members	5	4	4
Life Members	9	11	6
Honorary Members	0	1	1
Associate Members	0	1	0
	30	26	19

Summary:		1912–13.
Total Elections		291
Total Resignations	31	
Total Deaths	19	
	—	50
Net increase in membership		241

It will be noted that there has been a net increase in membership of 241, the net increase during the preceding year being only 43, showing a very substantial gain in membership. It is hoped that this increase can be continued during the coming year, as the improved work which the Institute is doing is attracting more general interest.

Respectfully submitted,

W. C. L. EGLIN,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON STOCKS AND FINANCE.

Financial Statement, October 1, 1912, to September 30, 1913.

PROPERTY AND FUNDS.

Building and land, 13–17 S. Seventh Street		$60,000.00	
Library		100,000.00	$160,000.00
	Principal.	Unexpended Income.	
Funds held by Board of Trustees	$264,528.48	$273.73	
Funds held by Board of Managers	23,478.50	62.96	
Franklin Institute Building Fund	371,185.16	....	
Elliott Cresson Medal Fund	1,000.00	2,082.50	
Franklin Fund and Building Committee.	886.59	....	
Total funds	$661,078.73	$2,419.19	$663,497.92
Grand total			$823,497.92

LIABILITIES.

Certificates of stock	$30,420.00
Mortgage on Institute Building (held by Trustees as investment for funds)	21,325.00
Bills payable	22,250.00
Accounts payable	5,096.41
Unearned income	1,494.24
Grand total	$80,585.65

INCOME AND EXPENSES APPLICABLE TO YEAR ENDING SEPTEMBER 30.

Income.

Dues		$9,740.31
Initiation fees		910.00
General Endowment Fund		9,375.82
James H. Cresson Memorial Fund		1,091.45
Estate of John Turner		129.15
Estate of Robert Wright		1,394.55
Miscellaneous		19.00
Membership badges		6.50
Instruction: Drawing	$1,414.50	
Mathematics	512.25	
Mechanics	307.24	
Naval Architecture	389.76	2,623.75
Publications: Subscriptions and sales	$1,817.55	
Advertising	2,779.65	4,597.20
Total		$29,887.73

Expenses.

Building: Wages	$1,153.50	
Repairs and maintenance	429.17	
Heat, light, and power	655.30	
Taxes, water rent, and insurance	581.68	
Miscellaneous supplies and expense	221.14	3,040.79
Elections and resignations		360.41
Instruction: Drawing	$1,145.62	
Mathematics	298.09	
Mechanics	203.11	
Naval architecture	233.23	
Popular lectures	109.19	
Miscellaneous expense	774.66	2,763.90
Library: Salaries	$3,763.43	
Books and periodicals	1,641.29	
Binding	663.00	
Miscellaneous expense	234.44	6,302.16
Meetings		494.45
Office and General: Salaries	$8,191.14	
Office expense	501.66	
General expense	1,269.87	9,962.67

Publications:	Printing	$5,543.25	
	Reprints	248.32	
	Illustrating	661.16	
	Miscellaneous expense	465.48	
	Year-Book	497.02	7,415.23
Science and Arts			1,382.68
Sections			939.05
Interest and discount			1,497.08
Total			$34,158.42
Net expense			4,270.69

During the fiscal year there has been an increase in funds as follows:

General Endowment Fund	$225.00
James H. Cresson Memorial Fund	50.000.00
Edward Longstreth Medal Fund	1,000.00
Elizabeth M. Graff Fund	14.38
Franklin Institute Building Fund	11,104.94
Franklin Fund and Building Committee	68.29

The mortgage on the Institute building has been decreased by $750.00.
Bills payable have been increased by $5,000.00

The exc ss of expense over income for the year was $4,270.69.

Respectfully submitted,

WALTON FORSTALL,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON PUBLICATIONS.

To the Board of Managers:

The principal publications in charge of this committee are the JOURNAL of the Institute and its Year-Book. The contents of the JOURNAL have been maintained during the past twelve months on the high plane of excellence to which these had been advanced during the preceding year. Through the collaboration of the corps of associate editors organized in 1912, the JOURNAL has been brought to occupy again that conspicuous place as a chronicle of progress in science and the arts which it held with such marked distinction during the greater part of the 19th century. This distinction became gradually diminished as the journalistic field of practical technology and of the various industrial arts became increasingly occupied by publications devoted to individual trades, and by those of specifically professional organizations. The very growth, however, of these specialized publications, whose scope can at best but scarcely extend beyond the range of daily practice, has resulted in leaving a larger place for a chronicle of original research in the application of science in the arts, and this place our distinguished collaborators are efficiently helping us to fill.

Of those whose help in this direction we obtained at the beginning, eighteen months ago, two, namely Dr. Weston and Dr. Whitney, found it

necessary to relinquish the work, their places being taken by Dr. Wilder D. Bancroft, Professor of Physical Chemistry, Cornell University, and Dr. W. J. Humphreys, Professor of Meteorological Physics, U. S. Weather Bureau. Our esteemed colleague, Mr. Edwin S. Balch, having, after many years of helpful service on this Committee, resigned his membership, Dr. George A. Hoadley was appointed to fill the vacancy.

The Year-Book of the Institute, like the JOURNAL, has been kept up to the standard latterly set for it, as regards not only the completeness and accuracy of its contents, but its mechanical make-up as well. In both respects this publication now properly reflects the dignity of The Franklin Institute.

The financial aspects of the committee's work may be briefly summarized as follows:

The receipts from all sources, including the credits due the publication account for the additions to the library through exchanges of the JOURNAL for other scientific periodicals and exchanges of advertising space for books, amounted to $6,493.05, and the expenditures to $6,356.53. The surplus of $136.52 is due principally to receipts from reprints, there having been a decrease of $146.60 in our income from other sources.

Respectfully submitted,

LOUIS E. LEVY,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON EXHIBITIONS.

To the Board of Managers:

The Committee on Exhibitions calls attention to the number of displays which have been features of large conventions, held in various portions of the country, and to the success of most of these, which may be interpreted as demonstrating a desire of the public for exhibitions.

The facilities for such displays in Philadelphia are limited, and the cost of a satisfactory building to accommodate an exhibition, such as The Franklin Institute should sponsor, would necessitate efforts to accumulate a fund which might interfere with securing money for the new Institute home.

The Committee, however, believes that when the buildings covering the site selected for the permanent housing of the Institute are razed, there may be available space which could be used for a temporary structure, or possibly some of the area allotted to Parkway can be secured for such purpose.

The Committee believes that an exhibition devoted to special features would be more desirable than to attempt a general exposition, but at the present time has no recommendation to make.

It will, however, keep advised as to the progress made for the Panama Exposition, having in mind the possibility of securing some of the exhibits, if the time immediately preceding or following the California enterprise coincides with opportunity to secure available accommodations in Philadelphia.

Respectfully submitted,

JOHN BIRKINBINE,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON SECTIONAL ARRANGEMENTS.

To the Board of Managers:

The Committee on Sectional Arrangements begs to report that during the year ending September 30, 1913, sixteen lectures were given by men of national prominence, covering a wide range of subjects of unusual interest and great practical value.

The average attendance was slightly better than that for the preceding year. Following is the list of lectures:

PHYSICS AND CHEMISTRY SECTION.

Seven Meetings.

October 3, 1912.
Industrial Research.
Robert Kennedy Duncan, B.A., Sc.D.; Director Industrial Research and Professor of Industrial Chemistry, University of Pittsburgh and University of Kansas; lecturer at Clark University.

November 7, 1912.
The Chemistry of Nutrition.
R. H. Chittenden, Sc.D.; Director Sheffield Scientific School, Yale University.

December 12, 1912.
Chemiluminescence.
Wilder D. Bancroft, Ph.D.; Professor of Physical Chemistry, Cornell University.

January 30, 1913.
The Physical Laboratory of the National Electric Light Association.
Edward P. Hyde, Ph.D., Director of Physical Laboratory N.E.L.A.

March 6, 1913.
Electricity and Matter.
Arthur W. Goodspeed, Ph.D., Professor of Physics, University of Pennsylvania.

March 13, 1913.
The Chemistry of Paper Making.
Arthur D. Little, Chemical Engineer; President Arthur D. Little, Inc.; Special Lecturer Massachusetts Institute of Technology.

April 10, 1913.
Colloids and Crystals—The Two Worlds of Matter.
R. H. Bradbury, Ph.D.; Head of Dept. of Science, Southern High and Manual Training School.

ELECTRICAL SECTION.

Two Meetings.

January 9, 1913.
The Electrical Propulsion of Ships.
W. L. R. Emmet, Sc.D.; Consulting Engineer and Engineer of Lighting Department U. G. I.

April 3, 1913.
The Electrical Equipment of a Modern Battleship.
H. A. Hornor, B.A.; Electrical Engineer New York Ship Building Company.

MECHANICAL AND ENGINEERING SECTION.

Five Meetings.

October 10, 1912.
The Testing of Full-Size Pieces under Practical Conditions, together with Locomotive Testing in the United States.
Cav. Gaetano Lanza, the Baldwin Locomotive Works.

October 24, 1912.
Safety of Life at Sea.
Mr. James Donald, Consulting Naval Architect, Fore River Ship Building Company.

December 5, 1912.
Recent Advances in the Steam Turbine Art.
Mr. H. T. Herr, V.P. and Gen. Mgr. Westinghouse Machine Company.

April 24, 1913.
Industrial Combustible Gases.
Mr. J. M. Rusby, Engineer of Tests, United Gas Improvement Company.

May 1, 1913.
The Design of Large Bridges with Special Reference to the New Quebec Bridge.
Ralph Modjeski, D.Sc.; Consulting Engineer; American Member of the Commission which designed the New Quebec Bridge.

MINING AND METALLURGICAL SECTION.

Two Meetings.

November 14, 1912.
Experimenting with the Electric Furnace.
F. A. J. FitzGerald, A.B., B.S.; FitzGerald & Bennie Laboratories.

February 13, 1913.
An Outline of the Metallography of Iron and Steel.
Albert Sauveur, B.S.; Professor of Metallurgy and Metallography, Harvard University.

All of the lectures, except Professor Duncan's, were illustrated by lantern slides and, in addition, practical demonstrations were presented by Dr. Bancroft and Dr. Bradbury.

Thirteen of the lectures have been published in the JOURNAL.

Dr. A. A. Michelson, of the University of Chicago; Dr. Harry F. Keller, of the Philadelphia High School, and Professor M. E. Cooley of the University of Michigan, were unable to be present upon the dates set for their lectures, but it is hoped that arrangements may be made to hear them during the coming year.

Respectfully submitted,

CHARLES DAY,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE ENDOWMENT COMMITTEE.

To the Board of Managers:

The Committee announces the death of Mrs. William H. Wahl, which occurred on December 16, 1913. According to the terms of the will of Dr. William H. Wahl, deceased, The Franklin Institute becomes the residuary legatee, providing that the Institute, within three years from the date of Mrs. Wahl's death, obtains, by subscription or by appropriation from the City of Philadelphia, or the State of Pennsylvania, or by gift from a source other than by bequest, a sum equal to the appraised value of the estate.

We have not, as yet, received a full account of the estate from the executors and trustees under the will, The Philadelphia Trust Company, but we have in our possession a memorandum which shows that the Real Estate is assessed at $25,660, and that the Personal Estate is assessed at $59,085, making a total of $84,745. We believe that it will be necessary for the Institute to raise at least $90,000 within three years in order to receive this bequest.

This bequest is to be known as the "John H. Wahl Fund" in memory of Dr. Wahl's father.

At a meeting of the Endowment Committee, January 7, 1914, it was decided to at once prepare and send a letter to each member of the Institute, setting forth this bequest and asking for subscriptions to help meet the requirements of the will. One subscription has already been received.

The Endowment Committee takes pleasure in announcing that Mr. Samuel Insull, of Chicago, Illinois, has given the sum of $6,000, which is to be known as "The Franklin Medal Fund, founded January 1, 1914, by Samuel Insull;" $1,000 of this amount is to be used for securing the proper design and the necessary dies and diplomas. The remaining $5,000 is to be invested and is to be used in awarding the Franklin Medal to those workers in Physical Science and Technology, without regard to country, whose efforts have, in the judgment of the Managers of the Institute, been most productive in advancing Physical Science and its applications.

The committee also announces the generous gift by Miss Mary Thorn of one thousand dollars for the establishment of scholarships in the School and three hundred dollars for a Permanent Resident Membership.

Respectfully submitted,

HENRY HOWSON,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON LIBRARY.

(*Abstract.*)

To the President and Members of The Franklin Institute:

During the twelve months ending September 30, 1913, the contents of the library were increased by 3306 additions, this number being 335 more than the additions of the preceding year. The new accessions were derived as follows:

SOURCE.	Bd. Vols.	Unbd. Vols.	Pphs.	Maps.
Gift	439	528	1555	1
Exchange account	25	5		
JOURNAL	28	36	46	
By purchase:				
Institute appropriation	36	11	1	
Lea Fund	4	2		
Moore Fund	102	15	1	
Morris Fund	23	2		
Potts Fund	14	5		
Binding:				
Institute Appropriation	405			
Chemical Periodical Binding Fund	22			
	1098	604	1603	1

Gifts of books, pamphlets, and periodicals in large quantities or of special value were received as follows:

Mr. H. M. B. Bary, copies of engineering publications.

Mr. George S. Cullen, volumes of the *Jewellers' Circular* and other publications.

Dr. E. V. d'Invilliers, books, pamphlets, and magazines on geology, mining, etc.

Mr. Spencer Fullerton, pamphlets and magazines on various subjects.

Mr. F. Lynwood Garrison, volumes and pamphlets on mining and related subjects.

Mr. Richard Gilpin, volumes and pamphlets on electrical subjects.

Mr. J. A. Given, copies of the *Electrical Engineer*, *Electrical World*, *American Electrician*, and *Electrician*.

Mr. H. B. Hackett, books and pamphlets on electricity and engineering.

Mr. Robert R. Kitchel, copies of *Iron Trade Review* and *Engineering Record*.

Mr. Thomas M. Lightfoot, books pamphlets, and magazines on engineering subjects.

Mr. Richard Waln Meirs, eight bound volumes.

Mr. E. H. Sanborn, collection of clippings on iron and steel taken from newspapers and magazines, classified and arranged in filing cases.

Mr. Wm. R. Webster, volumes of publications of engineering societies.

Mrs. Martha C. Heckendorn, volumes from the library of the late Henry C. Heckendorn.

The Baldwin Locomotive Works, U. S. Patent Office Reports.

Henry Bower Chemical Manufacturing Company, chemical publications and patent reports.

The Engineers' Club of Philadelphia, engineering periodicals and magazines of the year 1912.

Messrs. Harris & Richards, books, pamphlets, and magazines on engineering and architectural subjects.

Koenigliche Technische Hochschule zu Muenchen, fifty-five pamphlets consisting of dissertations and other publications.

The committee is again indebted to Mr. Joseph A. Arnold, Editor and Chief, Division of Publications, U. S. Department of Agriculture, for supplying the library with copies of the current publications of the department.

NON-TECHNICAL MATERIAL AND PUBLIC DOCUMENTS.

The year's accumulation of duplicate public documents and non-technical publications was transferred to the Free Library. The material so disposed of amounted to 110 volumes and four pamphlets.

CONTENTS OF THE LIBRARY.

After deducting the duplicate books withdrawn and discarded during the year, the library contained, at the close of the Institute year on September 30th, 1913:

Volumes, bound and unbound	64,169
Pamphlets	48,648
Maps and charts	2,272
Photographs	1,336
Newspaper clippings	192
Manuscripts	34

BINDING.

The following bookbinding was done for the library:

	Bound	Repaired	Rebound
Recent volumes of periodicals	405		
Recent volumes of periodicals charged to Chemical Periodical Binding Fund	22		
Old volumes	132	21	3
Books purchased with the income of the Lea Fund	15		
Memorial Library Fund	2		
Moore Fund	31		
Morris Fund	3		
Potts Fund	2		
Books received from the JOURNAL	3		
	615	21	4

REMARKS.

Additions.

The continued increase in the accessions is due in a large measure to the systematic efforts which are being made to obtain all technical publications of which notices appear in the current magazines.

The committee is indebted to the donors above mentioned for their contributions.

List of Serials.

Among the special work undertaken during the year is the preparation of a list, on cards, of the status of the current periodical sets in the library. This list will present an accurate statement of the condition of the file of each periodical—when first issued, how many volumes have appeared, and, if incomplete, what parts are missing. The progress on this work is slow because of the care required in its preparation and the limited time which can be devoted to it. It is, however, expected that in the course of another year this list of periodical publications will be finished. When this is done, such sets as have ceased publication or which are no longer received by the library will be listed in the same manner.

Pamphlets.

The pamphlets on mechanical subjects are being kept together, and when a sufficient number on one subject are accumulated these will be bound and properly placed on the book shelves.

It is the intention of the committee to have the remaining pamphlets, which are still stored, gone over and classified according to subjects; each class to be treated as were those relating to mechanics. In this way the entire accumulation of pamphlets, many of which are of considerable importance, will eventually be converted into bound volumes, duly catalogued and made accessible.

Stack Room.

The stack room is again very much crowded, but the shelving in the basement, to which reference was made in last year's report, will shortly be ready, and some of the periodical sets which are rarely consulted will be removed to those shelves. It is estimated that about 3200 volumes can be accommodated in the basement rooms, and this will greatly relieve the present overcrowding in the stack.

LIBRARY PUBLICATIONS.

Bulletin No. 3, which will contain the additions to the library during the past two years, is now in course of preparation and will shortly be printed and published.

Respectfully submitted,

LOUIS E. LEVY,
Chairman ad int.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON MUSEUMS.

To the President and Members of The Franklin Institute:

The Committee on Museums begs respectfully to report that during the year now closing quite a number of interesting and valuable gifts have been added to the collection of models, memorabilia, etc., partly owing, no doubt, to the revived interest shown in the collection since the completion of the onerous task of bringing out, repairing, classifying, and displaying the large

number of articles that had been stored away out of sight for years. A brief account of this work of resuscitation was given in the last annual report of this Committee.

The Institute's collection of models has been taken good care of during the year and the additional models have been accommodated as well as possible.

To display the models to their best advantage would require much more space than is now available. It is expected that with the maturing of plans now in prospect for providing the Institute with a new building, ample provision will be made for the Institute's Museum; meanwhile an endeavor has been made to house the models and preserve them.

A list of gifts received during the year and placed in charge of the committee is appended.

Respectfully submitted,

ALEX. E. OUTERBRIDGE, JR.,
Chairman.

PHILADELPHIA, January 14, 1914.

APPENDIX.

Gifts to the Institute during the year October 1, 1912, to September 30, 1913. Placed in charge of the Committee on Museums.

Nitrogen-filled Tungsten Lamp, 110 volts, 1000 C. P. Gift of Mr. J. W. Lieb, Vice-President, The New York Edison Company.

Edison Z-Type Dynamo. 17.6 K. W., 110 volts. Gift of The William Cramp and Sons Ship and Engine Building Co. This machine was in service in the power house of the company until the spring of 1913.

Herrick Balanced Rotary Engine, 110 H. P., designed and developed by Mr. Gerard P. Herrick, President Gerard Development Company. Gift of the designer.

Deed, dated 1787, bearing an excellent signature of Benjamin Franklin. Gift of Mr. Lawrence T. Paul.

Apparatus used in experiments on Vortex Motion in Liquids by Dr. E. F. Northrup, Princeton University. Gift of the experimenter.

Edison Three-Wire Chemical Meter, No. 102. Gift of Mr. G. L. Thompson, General Electric Company.

Cell used by Mr. Samuel Rea and Mr. Jesse Lightfoot in original experiments in which willow charcoal was made to glow with electric current. Gift of Mr. Thomas M. Lightfoot.

Framed photograph of the late Joseph M. Wilson, President of the Institute, 1887 to 1896. Gift of Mr. Wilson's brother, Mr. Henry W. Wilson.

Collection of Alundum Refractories. Gift of the Norton Company.

Medals granted to Mr. J. E. Wootten (Paris Exposition, 1878; John Scott Legacy Medal, 1891; National Exposition of Railway Appliances, 1883, presented by Philadelphia and Reading Railroad 1884). Gift of Mrs. Augustus W. Hoff.

Framed photograph of Mr. John E. Wootten. Gift of Mrs. Augustus W. Hoff.

Case of Electrical Fittings. Gift of the Bryant Electric Co.

Early form of Steam Engine Indicator, invented by Wm. Macnaught, Glasgow, and used on steamboat "Richard Stockton." Gift of Mr. W. F. Carnes, Harlan and Hollingsworth Corporation.

Hirsch Electric Cap Lamp for use in mines. Gift of the Hirsch Electric Mine Lamp Co.

First Model of Ellis Adding Typewriter. Gift of Mr. Halcolm Ellis.

Autograph letter from John Tyndall, Royal Institution of Great Britain, dated June 12, 1882, to A. E. Outerbridge, Jr., referring to "a very important observation made by General Duane on the Coast of the United States," and autograph letter from Sir Frederick Bramwell, dated London, December 4, 1882, to A. E. Outerbridge, Jr., referring to "Sterlings Toughened Cast Iron," and enclosing patent specification No. 11,262 issued in 1846. Gifts of Prof. A. E. Outerbridge, Jr.

REPORT OF COMMITTEE ON MEETINGS.

To the President and Members of The Franklin Institute:

During the year ending September 30, 1913, eight stated meetings were held.

At the May meeting the Institute's Elliott Cresson Medal was presented to Dr. Chas. P. Steinmetz, Dr. Isham Randolph, and Mr. Emile Berliner. The presentations were made by President Walton Clark, and were followed by addresses from each of the recipients of the medal.

Under arrangements made by the Committee on Meetings, with the coöperation of the Secretary's office, papers were presented at the meetings as follows:

October 16, 1912: "Recent Developments in Radiography." Mr. H. Clyde Snook.

November 20, 1912: "The Railway Tunnels of New York City." Dr. Alfred Noble.

December 18, 1912: "The Engineer in the Building of the Republic." Dr. Isham Randolph.

January 15, 1913: "Applications of the Gyroscope to Marine Purposes." Mr. Elmer A. Sperry.

February 19, 1913: "Transmission of Electrical Energy." Prof. Elihu Thomson.

March 19, 1913: "The Generation and Distribution of Energy." Mr. Samuel Insull.

April 16, 1913: "The Wireless Telephone." Prof. G. W. Pierce.

May 21, 1913: "Development of the Talking Machine." Mr. Emile Berliner.

"The Imaginative Faculty in Engineering." Dr. Isham Randolph.

"Some Electrical Problems Awaiting Solution." Dr. Chas. P. Steinmetz.

These papers were referred to the Committee on Publications, and afterwards, with the exception of one, were published in the JOURNAL.

JAMES S. ROGERS,
Chairman.

PHILADELPHIA, January 14, 1914.

REPORT OF THE COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract.*)

To the President and Members of The Franklin Institute:

I am glad to report that the work of the Committee on Science and the Arts for the past year has been excellent. All meetings of the Committee have been well attended and a larger number of cases have been examined and reported upon than for a number of years. The result has been that more honors than usual have been conferred by the Institute.

As a further means, however, of increasing the effectiveness of the Committee's work, the following suggestions are respectfully submitted:

First.—That the work of the Committee be made more generally known to the Institute membership and others interested in the Institute's work.

Second.—That the services of specialists, non-members of the Committee among the Institute's membership and among scientific people generally be sought to assist the Committee in its work. While its membership is large, the extent of the field covered by the work of the Committee is so great that difficulty is sometimes experienced in naming a sub-committee who have had the necessary experience in relation to the subject of the application.

Third.—That the work of the sub-committees on Preliminary Examination and on New Subjects and Publicity be merged into one committee, consisting of ten members, and known as the Sub-committee on New Subjects and Preliminary Examination, and that this Committee address the Institute membership and others through the Secretary of the Institute concerning the work of the Committee and asking for information relating to the work of meritorious inventors.

Fourth.—That as fewer people than desired in this country and abroad are familiar with the Institute's work in stimulating invention and discovery in physical science, it is thought that the Secretary of the Institute might arrange with the various principal scientific and technical journals to give a wider publicity to the work of this Committee, to the end that the Institute may render further assistance to inventors and scientists.

Fifth.—That an effort be made to reach the younger and less well-known members of the body of inventors and scientific workers; though their work may not entitle them to the highest awards of the Institute, yet much of it is meritorious and might be recognized by our Certificate of Merit. In order to attain a knowledge of such improvements and minor advances in the mechanic arts, which though small individually in the aggregate materially assist in the development of these arts, our present methods of obtaining such information will have to be extended. It is suggested that the Secretary of the Institute prepare a circular letter, to be sent to engineering firms, manufacturing establishments, and public service corporations, describing the work of this Committee and asking for the names of deserving young men who have invented (not necessarily patented) novel and useful devices.

Sixth.—That the Secretary of the Institute prepare each year a summary of the awards which have been made during the year so that the value of the

work of the Institute through its Committee on Science and the Arts in promoting invention and scientific discovery may be more readily estimated.

Seventh.—That as a means of bringing the members of the Committee into closer social relationship and as furnishing an opportunity for the informal interchange of ideas, arrangements be made for a simple Science and Arts dinner prior to a meeting of the Committee.

As retiring Chairman I hope, Mr. President and gentlemen, that you will allow me to say to my fellow members that I have derived both pleasure and profit from my connection with them in their work of the past year.

Respectfully submitted,

J. A. P. CRISFIELD,
Chairman.

PHILADELPHIA, January 14, 1914.

APPENDIX TO REPORT OF CHAIRMAN OF THE COMMITTEE ON SCIENCE AND THE ARTS.

STATEMENT OF COMMITTEE'S OPERATION
FOR THE
TWELVE MONTHS ENDING SEPTEMBER 30, 1913.

Cases.

Cases pending, October 1, 1912		5
Applications during the year	16	
Other cases opened	24	40
		45
Disposed of during the year		27
Leaving pending October 1, 1913		18

Disposition of Cases.

Elliott Cresson Awards	7
Edward Longstreth Awards	5
Howard N. Potts Awards	1
John Scott Legacy Recommendations	4
Certificates of Merit	0
Advisory	10
	27

DETAILS OF AWARDS.

ELLIOTT CRESSON MEDAL.

Emile Berliner, in recognition of important contributions to telephony and to the science and art of sound reproduction.

Emil Fischer, in recognition of numerous contributions of fundamental importance to the science of organic and biological chemistry.

Isham Randolph, in recognition of distinguished achievement in the field of civil engineering.

Sir William Ramsay, in recognition of numerous discoveries of far-reaching importance in the science of chemistry.

Lord Rayleigh, in recognition of extended researches of signal importance in physical science.

Albert Sauveur, in recognition of his numerous and important contributions to the science of metallography.

Charles Proteus Steinmetz, in recognition of successful application of analytical method to the solution of numerous problems of first practical importance in the field of electrical engineering.

HOWARD N. POTTS MEDAL.

William A. Bone, for his paper on "Surface Combustion," published in the JOURNAL.

EDWARD LONGSTRETH MEDAL OF MERIT.

Cleveland Abbe, for his paper on "The Obstacles to the Progress of Meteorology," published in the JOURNAL.

E. Leon Chaffee, for his paper on "A New System of Impact Excitation of Continuous Electrical Oscillations," published in the JOURNAL.

Harry C. Jones, for his paper on "The Nature of Solution," published in the JOURNAL.

I. N. Knapp, for his paper on "Natural Gas, with Incidental Reference to other Bitumens," published in the JOURNAL.

John Stone Stone, for his paper on "The Practical Aspects of the Propagation of High-Frequency Electric Waves along Wires," published in the JOURNAL.

JOHN SCOTT LEGACY MEDAL AND PREMIUM.

Awarded by the City of Philadelphia on the recommendation of The Franklin Institute.

Milton A. McKee, for his Process of Treating Printing Plates.

Harley Clifford Alger for his Liquid Measurer.

Halcolm Ellis, for his Adding Typewriter.

C. Francis Jenkins, for his Motion Picture Apparatus.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of the Stated Meeting held Wednesday, January 7, 1914.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, PA., January 7, 1914.

MR. J. A. P. CRISFIELD *in the Chair.*

The following reports were presented for first reading:

No. 2524.—Sperry's Gyro-Compass.

No. 2530.—Stumpf's Una-Flow Engine.
No. 2534.—Reno's Escalator.
No. 2542.—Batdorf Coin Counting and Wrapping Machine.
No. 2544.—Draeger's Pulmotor. Advisory. Adopted.
No. 2587.—Spielman's Cloth Cutting Machine.

SECTIONS.

Section of Physics and Chemistry.—A stated meeting of the section was held in the Hall of the Institute on Thursday, January 8, 1914, at 8 o'clock P.M., with Dr. George A. Hoadley in the chair.

C. G. Abbott, S. M., of the Smithsonian Institution, Washington, D. C., delivered an address on "Physics of the Sun—A Review of Recent Solar Research." In the lecture especial stress was laid on the study of the solar radiation by means of the spectroscope, spectroheliograph, bolometer, and radiometer. The work of this nature conducted by the Astrophysical Laboratory of the Smithsonian Institution in Washington and elsewhere in the United States, and also in northern Africa, was described at length. The lecture was copiously illustrated with lantern slides of solar phenomena and of astrophysical laboratories and apparatus. The paper was discussed, and the meeting adjourned.

JOSEPH S. HEPBURN,
Secretary.

Mechanical and Engineering Section.—A meeting of the section was held in the Hall of the Institute on Thursday evening, January 15, at 8 o'clock. Mr. G. R. Henderson occupied the chair.

The minutes of the previous meeting were approved as published. The Chairman introduced Mr. Rudolph Hering, Consulting Engineer and Sanitary Expert, New York City, who delivered a lecture on "Sewage Treatment."

Mr. Hering presented the modern experiences and views on the problems of collecting and treating the sewage of cities. Mr. Hering described various methods of purifying fluid sewage in septic tanks, coarse grain and sand filters, and also Imhof tanks. He then described the methods of treating solid sewage. His remarks were illustrated by lantern slides.

At the close of the lecture the subject was discussed by several of the members present. The thanks of the meeting were extended the speaker.

Adjourned.

WILLIAM E. BULLOCK,
Acting Secretary.

MEMBERSHIP NOTES.

Elections to Membership.

(*Stated Meeting, Board of Managers, January 14, 1914.*)

RESIDENT.

MR. A. F. KUNBERGER, 1706 North Broad Street, Philadelphia, Pa.
MR. R. A. MILLAR, Westinghouse Machine Company, Philadelphia, Pa.
MR. MARSHALL S. MORGAN, 1401 Arch Street, Philadelphia, Pa.

NON-RESIDENT.

MR. WM. I. BALLENTINE, 31 The Cambridge, Penn and Michigan Streets, Indianapolis, Ind.
DR. ELMER L. CORTHELL, North Egremont, Mass.
MR. J. E. GREINER, Fidelity Building, Baltimore, Md.
MR. D. W. LUM, Southern Railway Company, 1300 Pennsylvania Avenue, Washington, D. C.
MR. L. P. SHARPLES, West Chester, Pa.
MR. R. JOHN TITZEL, 40 Wall Street, New York, N. Y.
MR. J. A. L. WADDELL, 1012 Baltimore Avenue, Kansas City, Mo.
MR. FRED H. WAGNER, Bartlett Hayward Company, Baltimore, Md.

ASSOCIATE.

MR. RICHARD GREENWOOD, 4647 Leiper Street, Frankford, Philadelphia, Pa.

Changes of Address.

MR. C. W. BARWIS, P. O. Box 286, Gallitzin, Pa.
MR. W. E. FIRTH, Midvale Steel Company, Box 1606, Philadelphia, Pa.
MR. ANDREW W. HILL, 1917 South Broad Street, Philadelphia, Pa.
MR. W. N. JENNINGS, 113 A Commercial Trust Building, Philadelphia, Pa.
MR. H. J. KALTENTHALER, 625 West Upsal Street, Germantown, Philadelphia, Pa.
MR. EARL B. SMITH, U. S. Office of Public Roads, Fourteenth and B Streets, S. W., Washington, D. C.
MR. EDWIN D. TUCKER, 308 West One Hundredth and Fourth Street, New York City, N. Y.

NECROLOGY.

Prof. William Dennis Marks, consulting engineer and statistician, New York.

Mr. Theodore H. Morris, 1608 Market Street, Philadelphia, Pa.

Mr. Wallace B. Riegner, Engineer of Bridges, Philadelphia & Reading Railway, Philadelphia, Pa.

Mr. William Tatham, 1811 Walnut Street, Philadelphia, Pa.

LIBRARY NOTES.

Purchases.

American Annual of Photography, vol. 28, 1914.
AMOS, P. A.—Processes of Flour Manufacture. 1912.
BEST, W. W.—Science of Burning Liquid Fuel. 1913.
British Association for the Advance of Science.—Reports of the Committee on Electrical Standards. 1913.
BURR, W. H.—Suspension Bridges, Arch Ribs, and Cantilevers. 1913.

CHRISTIAN, M.—Disinfection and Disinfectants. 1913.
CLASSEN, A.—Quantitative Analysis by Electrolysis. Ed. 5. 1913.
COHEN, J. B.—Organic Chemistry. 2 volumes. 1909, 1913.
COHNHEIM, O.—Chemie der Eiweissköper. 1911.
CROSS, C. F., and BEVAN.—Researches on Cellulose. 2 volumes. 1895–1905; 1906 and 1907.
DEINHARDT, K., and A. SCHLOMANN.—Illustrated Technical Dictionary in Six Languages. Volumes 5–11. 1909–1913.
DREYER, J. L. E., comp.—Scientific Papers of Sir Wm. Herschel. 2 volumes. 1912.
Geologisches Zentralblatt.—General Register. Volumes 1–15. No date .
HAAS, P., and T. G. HILL.—Chemistry of Plant Products. 1913.
HALSEY, F. A.—Handbook for Machine Designers and Draftsmen. 1913.
HEATH, T.—Aristarchus of Samos, the Ancient Copernicus. 1913 .
HOUGH, R. B.—American Woods. Part 13. 1913.
INGERSOLL, L. R., and O. J. ZOBEL.—Mathematical Theory of Heat Conduction. 1913.
Jahrbuch für Photographie. 1913.
JUNKERS, H.—Investigations and Experimental Researches for the Construction of my Large Oil Engine. No date.
KREMANN, R.—Application of Physico-chemical Theory. 1913.
LEA, M. C.—Kolloides silber und die Photohaloide. 1908.
London Chemical Society.—Annual Reports of Progress. Volumes 1–9. 1904–1912.
LUEPPO-CRAMER.—Kolloidchemie und Photographie. 1908.
Photograms of the year. 1913.
REDWOOD, B.—Petroleum. 3 volumes. 1913. 3rd edition.

Gifts.

American Institute of Mining Engineers, Transactions, vol. 44. 1912. New York, 1913. (From the Institute.)
American Society of Mechanical Engineers, Condensed Catalogues of Mechanical Equipment. December, 1913. New York, 1913. (From Dr. R. B. Owens.)
Amsterdam Koninklijke Akademie van Wetenschappen: Proceedings of the Section of Sciences, vol. 15, pts. 1 and 2. Verhandelingen, deel 11, Nos. 5 and 6, and Jaarboek, 1912. Amsterdam, 1913. (From the Akademie.)
Baylor University, Annual Report of the President, 1912-1913. Waco, Texas, 1913. (From the University.)
Dartmouth College, Catalogue 1913–1914. Hanover, N. H., 1913. (From the College.)
Electrical Blue Book, 6th edition, 1913. Chicago, 1913. (From Dr. R. B. Owens.)
Harvard University, Catalogue 1913-14. Cambridge, Mass., 1913. (From the University.)
Haverhill Public Library, 38th Annual Report, 1912. Haverhill, Mass., 1913. (From the Library.)

Illinois State Water Survey, Bulletin No. 9, Chemical and Biological Survey of the Waters of Illinois. Urbana, 1912. (From the Survey.)

Institution of Naval Architects, Transactions, vol. 55, pt. 2. 1913. London, 1913. (From the Institute.)

John Crerar Library, Handbook, 1913. Chicago, 1913. (From the Library.)

Lake Mohonk Conference of Friends of the Indian, Report of the 31st Annual Conference. Mohonk Lake, N. Y., 1913. (From the Conference.)

Massachussetts State Board of Health, 44th Annual Report. Boston, 1913. (From the Board.)

New Hampshire Public Service Commission, Report 1912, vol. 2. Concord, no date. (From the Commission.)

New South Wales, Vital Statistics for 1912. Sydney, 1913. (From the Government Statistician.)

New York State Public Service Commission for the First District. Annual Report 1912, vol. 1. New York, 1913. (From the Commission.)

Ohio Department of Mines, Report of the Coal Mining Commission, 1913. Columbus, 1913. (From the Department.)

Ontario Department of Agriculture: 34th Annual Report of the Ontario Agricultural and Experimental Union, 1912, and Annual Report of the Bureau of Industries, 1912. Toronto, 1913. (From the Department.)

Pennsylvania Topographic and Geologic Survey Commission: Report for 1910–1912, and Report No. 6, Graphite Deposits of Pennsylvania, by Benjamin L. Miller. Harrisburg, 1912. (From the State Geologist.)

Philadelphia Department of Public Works, Annual Report of the Department and Bureau of Water, 1912. Philadelphia, 1913. (From the Department.)

Polytechnic Institute of Brooklyn, Catalogue of the College of Engineering, 1914–1915. Brooklyn, no date. (From the Institute.)

R Instituto d' Incoraggiamento di Napoli, Atti, Serie Sesta, 1912. Napoli, 1913. (From the Instituto.)

St. Louis & San Francisco Railroad Company, 17th Annual Report, 1913. St. Louis, no date. (From the Company.)

Simeon North, First Official Pistol Maker of the United States: A Memoir, by S. N. D. North and Ralph H. North. Concord, N. H., 1913. (From R. H. North.)

Southern Pacific Company, 29th Annual Report. New York, 1913. (From the Company.)

Stephens Memorial Library, Brief Subject Catalogue, 1913. Philadelphia, 1913. (From the Librarian.)

United States Library of Congress, Report 1913. Washington, D. C., 1913. (From the Library.)

University of Chicago, Annual Register, 1912-1913. Chicago, 1913. (From the University.)

Vassar College, 49th Annual Catalogue, 1913–1914. Poughkeepsie, N. Y., 1913. (From the College.)

Vermont State Library: Vermont Insurance Report, 1912, and Bank Commissioner's Report, 1913. Montpelier, 1913. (From the Library.)

Victoria Memorial Museum, Bulletin, No. 1. Ottawa, 1913. (From the Canada Geological Survey.)

Worcester Polytechnic Institute, 44th Annual Catalogue, 1913–1914. Worcester, Mass., 1913. (From the Institute.)

Yonkers Bureau of Water, 40th Annual Report, 1913. Yonkers, N. Y., 1913. (From the Bureau.)

PUBLICATIONS RECEIVED.

The Mechanical Engineer's Reference Book. A Hand-book of Tables, Formulas, and Methods for Engineers, Students, and Draftsmen, by Henry Harrison Suplee, B.Sc., M.E. 4th edition, revised and enlarged. 964 pages, illustrations, 12mo. Philadelphia, J. B. Lippincott Company, 1913. Price, $5.

Le Chatelier, H.—Molécules, Atomes et Notations Chimiques. Mémoires de Guy-Lussac, Avogadro, Ampère, Dumas, Gaudin, Gerhardt. 116 pages, plate, 12mo. Paris, Armand Colin, 1913. Price, 1.20 francs.

Gautier, Henri.—L'air, l'Acide Carbonique et l'Eau. Mémoires de Dumas, Stas, Boussingault. 104 pages, plates, 12mo. Paris, Armand Colin, 1913. Price, 1.30 francs.

Industrial Organic Analysis for the Use of Technical and Analytical Chemists and Students, by Paul S. Arup, B.Sc., A.C.G.I., with a foreword by J. C. Irvine, D.Sc., Ph.D. 340 pages, illustrations, 12mo. Philadelphia, P. Blakiston's Son and Company, 1913. Price, $2.25.

Quantitative Chemical Analysis, Adapted for use in the Laboratories of Colleges and Technical Institutes, by Frank Clowes, D.Sc., Lond., and J. Bernard Coleman, A.R.C.Sc., Dublin. Tenth edition. 577 pages, illustrations, 8vo. Philadelphia, P. Blackiston's Son and Company, 1914. Price, $3.50.

Allen's Commercial Organic Analysis. A Treatise on the Properties, Modes of Assaying, and Proximate Analytical Examination of the Various Organic Chemicals and Products Employed in the Arts, Manufactures, Medicines, etc., with Concise Methods for the Detection and Estimation of their Impurities, Adulterations, and Products of Decomposition. Volume VIII: Enzymes, Proteins and Albuminoid Substances, Milk, and Milk Products, Meat and Meat Products, Hæmoglobin and Blood, Proteids, Fibroids, by the editors and the following contributors: E. F. Armstrong, S. B. Schryver, L. L. Van Slyke, Henry Leffmann, Cecil Revis, W. D. Richardson, J. A. Gardner, E. R. Bolton, G. A. Buckmaster, W. P. Dreaper, Jerome Alexander. Fourth edition, entirely rewritten, edited by W. A. Davis, B.Sc., A.C.G.I., and Samuel S. Sadtler, S.B. 696 pages, illustrations, 8vo. Philadelphia, P. Blackiston's Son and Company, 1913 Price, $5.

Chemistry, Inorganic and Organic, with Experiments, by Charles London Bloxam. Tenth edition, rewritten and revised, by Arthur B. Bloxam, F.I.C., and S. Judd Lewis, D.Sc., F.I.C. 878 pages, illustrations, 8vo. Philadelphia, P. Blakiston's Son and Company, 1913. Price, $5.50.

U. S. Geological Survey: Mineral Resources of the United States, Calendar Year 1912. Part II—Nonmetals. 1218 pages, illustrations, plates, maps, 8vo. Washington, Government Printing Office, 1913.

The Manufacture of Reinforced Concrete Poles, Piles and Pipe without the Use of Forms. Process of R. M. Jones. 27 pages, illustrations, plates, folio. Denver, Col., 1914.

Topographic and Geologic Survey of Pennsylvania: Report No. 6, Graphite Deposits of Pennsylvania, by Benjamin L. Miller, Professor of Geology in Lehigh University. 147 pages, illustrations, maps, 8vo. Harrisburg, State Printer, 1912.

La Mesure de l'Intensité des Courants Faibles et de Haute Fréquence par Béla Gáti. Extrait de L'Industrie Électrique No. 525, 10 November, 1913, p. 498. 4 pages, illustrations, quarto. Paris, 1913.

Mellon Institute of Industrial Research and School of Specific Industries: Smoke Investigation, Bulletin No. 5. The Meteorological Aspect of the Smoke Problem, by Herbert H. Kimball, Ph.D., Professor of Meteorology, United States Weather Bureau. 51 pages, map, 8vo. Pittsburgh, Pa., University of Pittsburgh, 1913.

Canada Department of Mines: The Production of Coal and Coke in Canada during the Calendar Year 1912, by John McLeish, B.A., Chief of the Division of Mineral Resources and Statistics. 40 pages, 8vo. Ottawa, Government Printing Bureau, 1913.

U. S. Bureau of Mines: Bulletin 66. Tests of Permissible Explosives, by Clarence Hall and Spencer P. Howell. 313 pages, illustrations, 8vo. Monthly Statement of Coal-mine Fatalities in the United States, November, 1913. 23 pages, 8vo. Technical Paper 50—Metallurgical Coke, by A. W. Belden. 48 pages, illustrations, map, 8vo. Technical Paper 56—Notes on the Prevention of Dust and Gas Explosions in Coal Mines, by George S. Rice. 24 pages, 8vo. Washington, Government Printing Office, 1913–1914.

U. S. Bureau of Standards: Technologic Papers, No. 29—Variations in Results of Sieving with Standard Cement Sieves, by Rudolph J. Wig and J. C. Pearson. 16 pages, 8vo. Reprint No. 210—Observations on Ocean Temperatures in the Vicinity of the Icebergs and in other Parts of the Ocean, by C. W. Waidner, Physicist, H. C. Dickinson, Associate Physicist, and J. J. Crowe, Assistant Physicist, Bureau of Standards. (From Bulletin of the Bureau of Standards, vol. 10.) 14 pages, illustrations, plates, 8vo. Washington, Government Printing Office, 1913.

Harrison Safety Boiler Works: Cochrane Engineering Leaflet No. 16, Cochrane Metering Heaters and Cochrane Independent Meters. 48 pages, illustrations, 8vo. Philadelphia, Harrison Safety Boiler Works, 1913.

Color Changes in Colloidal Gold. S. H. LONG. (*Proc. Univ. Durham Phil. Soc.*, v, 113.)—Electrical migration and other tests were made on red, blue, and purple colloidal gold solutions prepared by electrical disintegration by means of a high-frequency alternating arc. The results show that the red colloid moves to the cathode and is probably associated with hydroxyl ions; the particles are very small. The blue colloid moves to the anode and is probably associated with hydrogen ions. The red colloid is less stable than the blue, into which it is converted by the action of an electrolyte or of an electric field. The purple colloid is a mixture of the red and blue forms in variable proportions.

CURRENT TOPICS

Record Production of Portland Cement in 1913.—According to returns received by the United States Geological Survey up to January 12, 1914, it is estimated by Ernest F. Burchard that the quantity of Portland cement manufactured in the United States in 1913 was approximately 92,406,000 barrels, compared with 82,438,096 barrels in 1912, an increase of about 9,967,900 barrels, or 12 per cent. The estimated shipments of Portland cement during 1913 were 88,853,000 barrels, compared with 85,012,556 barrels in 1912, an increase of about 3,840,400 barrels, or 4.5 per cent. On account of a large surplus of production over shipments stocks of cement at the mills apparently increased more than 45 per cent., or from 7,811,329 barrels in 1912 to 11,375,000 barrels at the close of 1913. In 1913 the relations between production and shipments were the reverse of those for 1912, when shipments exceeded production. It may be necessary to revise considerably the estimates of stocks, but it is believed that these figures for production and shipments are very close to those that will be shown by complete returns from all producers.

Although few definite statements as to selling prices are at hand, it is evident that the average value per barrel was appreciably higher than in 1913. Increases of 10 to 25 cents a barrel are reported from several plants in the Central and Eastern States, but there were slight decreases reported from a few plants in the Rocky Mountain district.

Two new plants, both in Washington, were added to the list of producers during 1913.

"Coalescence" in Metallic Lead. H. BANCKE (*Int. Zeits. Metallog.*, ii, 243.)—The growth of certain crystals at the expense of others, or "coalescence," in the case of lead has been ascribed by Rosenhain to an electrolytic effect wherein the eutectic acts as an electrolyte. Bancke shows that coalescence of rolled lead takes place on immersion in dilute acids, such as 5 per cent. acetic acid or 5 per cent. alcoholic nitric acid, more rapidly than was stated by Ewing and Rosenhain. The coalescence is sometimes energetic, with formation of large crystals, the metal becoming brittle. It has not been found possible by this "etching" method to control the extent of coalescence. With lead-foil the action is very rapid, and lead tubes often show vigorous coalescence. Coalescence precedes and makes possible rapid corrosion of lead. The examination of corroded lead showed that the unattacked portions consisted of normal-sized grains.

Photochemical Properties of Tungstic Acid. A. F. WASILIEFF. (*J. Russ. Phys. Chem. Soc.*, xliv, 819.)—In the presence of various reducing agents, *e.g.* cellulose, and under the influence of sunlight, tungstic acid undergoes change with formation of a blue color. According to Sabanéef, the action of hydrochloric acid on sodium tungstate gives, not colloidal tungstic acid, as Graham said, but sodium meta-tungstate: $4Na_2WO_4 + 6HCl = Na_2O_4WO_3 + 6NaCl + 3H_2O$. The author's investigations on the influence of sunlight in presence of dextrose on the initial product of this reaction indicates that this product, which is sensitive to light, is probably not sodium meta-tungstate. The initial product undergoes change into one which shows no photochemical properties at a velocity coresponding with the equation of a reaction of the first order; a rise of temperature causes the reverse change.

The International Engineering Congress of 1915.—Under the auspices of the American Society of Civil Engineers, the American Institute of Mining Engineers, the American Society of Mechanical Engineers, the American Institute of Electrical Engineers, and the Society of Naval Architects and Marine Engineers, there will be convened at the Panama-Pacific International Exposition, in 1915, an International Engineering Congress.

Engineers in all parts of the world have signified their intention of attending, and the delegation from Great Britain promises to be large in point of numbers and great in distinction.

The presidents and secretaries of the five organizations above named, together with eighteen other members, constitute the official body which will assemble and conduct the Congress. Colonel George W. Goethals, Chief Engineer of the Isthmian Canal Committee, has consented to act as Honorary President of the Congress, and will be in attendance in San Francisco when the celebration is held commemorating the completion of the greatest engineering achievement in all history

The week of September 20–25, 1915, will be the time of the sessions of the International Congress, and the proceedings of the meetings will be preserved in permanent book form. These volumes will contain the papers contributed to the Congress by eminent engineers, and will embrace in their scope every line of engineering activity, giving the latest word of authority in the field of engineering.

The executive offices of the Congress are in the Foxcroft Building, San Francisco, Cal., where the following officers of the committee are located: W. F. Durand, chairman; W. A. Cattell, secretary-treasurer; E. J. Dupuy, executive secretary.

A tentative schedule of general sessions has been formulated, and the following sections will be considered: 1, The Panama Canal; 2, Waterways and Irrigation; 3, Railways; 4, Municipal Engineering; 5, Materials of Engineering Construction; 6, Mechanical En-

gineering; 7, Electrical Engineering; 8, Mining Engineering; 9, Naval Architecture and Marine Engineering; 10, Military Engineering; 11, Miscellaneous.

Each of these sections will hold independent sessions during the Congress. Such joint sessions as may be deemed advisable will also be called. On account of the convention of the International Congress of Electrical Engineers just prior to the International Engineering Congress, the activity of the latter in the field of electrical engineering assigned to Section 7 will be limited to consideration of the points of contact between electrical engineering and other engineering phenomena and work.

Development of Photographic Images. W. H. CALDWELL. (*Eng. Pat.* 14,744, 1911.)—This invention provides a process of developing exposed photographic films in such a way that the silver halide dissolved by the developer shall not be reduced instantaneously, as in the known developers, but shall have time to travel to all the "unit portions" of silver halide which have been affected by light, and shall be reduced to silver in contact with each of such "unit portions," thus effecting much better development in the case of over-exposed images. For this purpose the proposed developer is an aromatic hydrazine in conjunction with an alkali bisulphite in solutions acid to litmus; for example, equal molecular proportions of phenylhydrazine and sodium bisulphite may be used. As phenylhydrazine has an irritant action on the skin, it may be replaced by *o*- or *p*-tolylhydrazine, or *o*- or *p*-bromophenylhydrazine. By regulating the strength of the developer any desired gradation may be produced. This mode of development may be applied to the films or surfaces described in English patent 1689 of 1908 to produce results of the correct scale of density, however prolonged the exposure may have been.

Use of Liquid Nitrogen for Producing Temperatures down to —211°C. G. CLAUDE. (*Comptes Rendus*, clvii, 277.)—The ease is demonstrated with which, in the absence of liquid hydrogen, it is possible to attain in a few minutes the solidification temperature of nitrogen, viz., —210° C. The principle adopted is that when a current of air is rapidly passed through a liquefied gas, this is cooled below its normal ebullition point. The process employed is more convenient, rapid, and permits the use of smaller quantities of liquid nitrogen than evaporation *in vacuo*. The liquid nitrogen is contained in a D'Arsonval-Dewar flask into which the extremity of a copper worm dips. The worm is cooled in another vessel by liquid nitrogen and traversed by a current of hydrogen, which is bubbled through the liquid, thereby producing rapid cooling, *e.g.*, —200° C. after two minutes, —206° C. after six, and —210° C. after twelve minutes. The experimental conditions are given, which are simple and allow of working with an open vessel.

Paper-Making Materials in Java. H. G. HAVIK (*Papier Fabrikant*, x, 859.)—*Bamboo.*—The bamboo is very abundant in Java, the principal species being *apus* and *andong*. For a mill producing 1500 tons of cellulose from three-year stems an area of 420 hectares (1040 acres) would have to be planted. In the large towns bamboo is worth 10 to 20 gulden ($4.25 to $8.50) per ton of air dry weight. From the apus bamboo there is obtained 50 per cent. of half-stuff on digestion with 19 per cent. of sodium hydroxide, calculated, on dry weight, for six hours under three atmospheres pressure. From the andong bamboo, with 15 per cent. of soda under a pressure of 4½ atmospheres for eight hours, a yield of 49 per cent. of pulp was obtained. The pulps were not suitable for bleaching, but gave excellent unbleached paper; the length of the fibres ranges from 1.6 to 4.6 mm.; average, 2.7 and 3.0 mm.

Alang-alang Grass.—This grows in great abundance and spreads very rapidly, attaining a height of 1.5 metres in a period of five weeks. This yields only 29.3 per cent. of cellulose on digestion with 9 per cent. sodium hydroxide for five hours under five atmospheres pressure; the pulp was difficult to bleach. This is at variance with Cross and Bevan's results, who obtained 55 per cent. of a cellulose which bleached easily. Length of fibre, 0.5 to 2.5 mm.; average, 0.8 mm.

Maize Straw is obtained in Java, often as a second crop after rice; more is produced than is needed. After cutting out the knots, the straw was digested with 21 per cent. of soda under three to four atmospheres for five hours; the yield was 22 to 23.2 per cent. of pulp not easily bleached; the paper made was of excellent strength. Length of fibre, 1 to 3 mm.; average, 1.5 mm.

Albizzia Moluccana.—Sheets of paper were prepared from the cellulose pulp of this wood, and from the mechanical pulp, blending them with other Javan fibres. From the results it is concluded that Albizzia wood is a useful and valuable paper-making material.

Water-Gas Tar for Killing Weeds. ANON. (*J. Gas Lighting*, cix, 332.)—Experiments made by the Conestoga Traction Company, of Lancaster, Pa., upon 160 miles of railroad track, show that a sprinkling of water-gas tar is an effective weed killer, and prevents the growth of fresh weeds. About 3000 gallons of tar were sprinkled on each mile of track.

The F-Rays do not Exist. ANON. (*Sci. Amer.*, cix, No. 15, 279.)—The discovery of F-rays recently announced has been contradicted. According to very precise and detailed information published by the *Eclair*, it appears that the results obtained are not at all conclusive. The F-rays appear to be an illusion similar to that to which the scientific world was a victim when the physicist Blondlot, Professor at the Faculty of Nancy, announced the discovery of the N-rays.

Peat for Power Purposes. H. V. PEGG. (*Inst. Mech. Eng.*, July 31, 1912.)—A gas producer at Portadown supplying about 250 British horsepower has given good results when working with peat which has not been subjected to any preliminary treatment for reducing its moisture, which varied roughly from 18 to 70 per cent., according to weather conditions. The only difficulty experienced was in cleaning the gas from tar; but this was overcome by the use of an ample water spray using about 7 gallons of water per British horsepower per hour. The engine valves required cleaning about once a week, and the whole plant was cleaned at the same time, though it was possible to run the producer for three weeks. The tar does not find a ready sale, owing to the pyroligneous odor, which can not be removed either from the tar or the oils distilled therefrom. A saving of over 50 per cent. was effected in the fuel bill by running the factory on peat instead of coal, consuming about 50 tons of peat per week. From peat containing 44.6 per cent. of carbon, 5.42 per cent. of hydrogen, and 18.98 per cent. of moisture a gas was obtained of the following composition: carbon dioxide, 10.6 per cent.; carbon monoxide, 21.0; hydrogen, 13.0; methane, 3.7; total combustible, 37.7 per cent.; calorific value, 144.0 British thermal units.

Synthetic Formation of Coal. BERGIUS. (*J. Gasbeleucht.*, liv, 748.)—By means of a specially-constructed bomb, bored out of a block of steel, and heated in an electrical furnace, carbon and water were found to react at about 350° C., producing carbon dioxide and hydrogen in the proportions of 1 : 2 by volume. Peat and cellulose, heated in the same apparatus to temperatures of 250° to 340° C., produced a substance which on further continued heating at 340° C. yielded a material resembling natural bituminous coal both in appearance and composition. As in the natural formation of coal, gases were evolved, chiefly carbon dioxide and methane. The longer the duration of the experiment, and the higher the temperature, the higher was the percentage of carbon and the lower that of oxygen in the resulting coal. The speed of reaction increases with the temperature; 64 hours at 310° C. were equivalent to 8 hours at 340° C., and from this change in speed of reaction it is calculated that, at the ordinary temperature, the formation of natural coal must have required about eight million years.

Iodine in Java. ANON. (*Rev. Scient. Rep.*, xxv, 352.)—Iodine is derived in considerable quantity from the saline constituents of certain mineral springs in Java, where it occurs chiefly as magnesium iodide, and is precipitated from them by means of copper sulphate. One Javan spring is said to yield as much as 12 centigrammes of iodine from each litre of water. The output of iodine from the Dutch East Indies amounts to about 30 tons per annum. It might be much more if more capital were employed in the undertaking.

Copra and Margarine and the Rise in the Price of Cocoanut Oil. ANON. (*Times*, Oct. 20, 1913.)—The comparative scarcity of copra in view of the increasing demand for margarine has caused a great rise in its price. In every quarter where copra is bought and sold, and even in Japan, despite the popularity of the Soya bean, plants for the manufacture of cocoanut oil are being rapidly erected. To extract the oil, the copra is milled, the resultant meal steamed and formed into cakes, and the oil squeezed out by hydraulic pressure. This is done, not upon the spot, but by a limited number of refiners in Europe and America. Cocoanut oil has long been used in the manufacture of free lathering soaps, called "washers" in the trade, as well as for what are termed "cold process" and "marine" soaps. It also is used for the manufacture of candles and night lights. On the other hand, it was never used for edible purposes in civilized countries, owing to its insinuating cocoanut taste and the speed with which it becomes rancid. These faults have been eliminated by chemical means, and cocoanut oil finds its way to the table in many forms. Cocoanut lard is now manufactured on a very large scale, and the oil has taken the place of animal fats in the manufacture of margarine. In its most highly refined form, blended with milk, it appears on the Continent as actual butter. Since the methods of purification were discovered the price has risen from £12 per ton to £48. The kernels are the staple food of the people where the nuts grow, and, as their needs must be satisfied before there is any margin for export, there does not appear any likelihood of overproduction.

A New Type of Kinematograph Camera. K. PROSZYNSKI. (*Phot. Journ.*, liii, 99.)—The article deals first with the principles governing the production of kinematograph effects, chiefly with regard to the elimination of flicker, and includes a detailed description, with illustrations, of a new type of kinematograph camera, which may be used like a hand camera, without any supporting stand. To obviate any effect of vibration, the front of the camera contains a powerful gyrostatic wheel, thereby keeping the camera very steady in any desired position. The motive power for driving the film is obtained from a set of cylinders containing compressed air, which are fitted in the back of the camera. These supply power to an efficient air motor which runs the film and shutter mechanism. Excellent results were demonstrated at the time the paper was read before the Royal Photographic Society.

Continuity. O. LODGE. (*Chem. News*, cviii, 127.)—The author urges a belief in ultimate continuity as essential to science; regards scientific concentration as an inadequate basis for philosophic generalization; believes that obscure phenomena may be expressed simply if properly faced; and points out that the nonappearance of anything perfectly uniform and omnipresent is only what should be expected, and is no argument against its real substantial existence.

Physico-Chemical Studies of Photographic Developers: Quinol as an Inductor. N. SCHILOW and E. TIMTSCHENKO. (*Z. Elektrochem.*, xix, 816.)—Sodium arsenite has an effect on the atmospheric oxidation of quinol solutions very similar to that of sodium sulphite; during the earlier stages of the oxidation the process is less rapid than without arsenite, but in the later stages the total amount of oxygen absorbed is greater than when arsenite is absent. Since sodium arsenite is not likely to form complex compounds with quinol, it is probable that it acts merely as a protective agent for the quinol. The oxygen is distributed between the quinol and the arsenite in the rates of 1 : 1.2, as compared with 1 : 1, which is the ratio for an ideal "acceptor" of oxygen. Analogous experiments with the aid of hydroxylamine and hydrazine as "acceptors" gave somewhat similar results, but it is considered probable that series of addition products are formed in these cases. The retarding and "regularizing" effect of hydroxylamine is so marked that its use in the preparation of "stand" developers is suggested.

Chromium Red Glaze. C. E. RAMSDEN. (*Trans. Eng. Ceram. Soc.*, xii, 239.)—This communication deals with the best glaze, maturing at the desired temperature, which will develop and display the tint imparted by the stain. The presence of lime is essential to prevent decomposition of the stain by the glaze. Though lime in excess of 0.2 CaO is unnecessary its content should never drop below 0.1 CaO. The basic oxides should consist largely of lead oxide, which helps to form a translucent yellow glass imparting a blood-red tint to the crimson stain. A small amount of alkali oxide is useful to produce translucency, but the proportion R_2O : RO should not be greater than 1 : 4; the stain is soluble in alkaline silicate and borates; both potassa and soda impart a lilac or violet tint. Alumina destroys the crimson color if in greater proportion than $0.3Al_2O_3$. If not fritted with the glaze mixture, as much as $0.5Al_2O_3$ may be introduced to obtain a pink; any excess over $0.1Al_2O_3$ diminishes the red tint. Boric acid imparts translucency, while it has only a slight tendency to communicate a lilac shade; the most favorable quantity appears to be equimolecular with the lime content, the greater the lime the more boric acid being permissible without attacking the stain. Zinc oxide brightens the surface, and may be introduced up to half the molecular content of lime. More than 0.3 BaO destroys the color; 0.5 SrO impairs the tint; magnesia should be excluded entirely. Silica may vary from 1.5 to 2 equivalents, according to the time and temperature of firing; glazes rich in silica tend to acquire a green tint. The author arranges oxides into two classes, according to their influence on the crimson glaze at cone 0.5 and in the order of their apparent effect on the color, beginning with that which has the greatest effect: *Favorable*—CaO, Bi_2O_3, PbO, B_2O_3, SiO_2, Na_2O, K_2O, Al_2O_3. *Injurious*—Sb_2O_3, MgO, P_2O_5, SrO, ZnO, BaO, and Al_2O_3.

Approximate Melting-Points of Some Commercial Copper Alloys. H. W. GILBERT and A. B. NORTON. (*Amer. Inst. of Metals*, Oct., 1913.)—The alloys (except manganese bronze, in which case the previously made alloy was melted) were all made by melting together the constituents in the carborundum crucible in which the determinations were made. Heating and cooling curves were plotted, and the temperature at which solidification began or fusion was completed was taken as the melting-point. The composition given is that aimed at, and actual analysis, in all cases where it was made, showed that this was very nearly attained. The results are as follows:—

Alloy	Percentage composition				Melting-point, °C.
	Cu.	Zn.	Sn.	Pb.	
Gun-metal	88	2	10	—	995
Leaded gun-metal	85½	2	9½	3	980
Red brass	85	5	5	5	970
Low-grade red brass	82	10	3	5	980
Leaded bronze	80	—	10	10	945
Bronze with zinc	85	5	10	—	980
Cast yellow brass	67	31	—	2	895
Naval brass	61½	37	1½	—	855
Manganese bronze	56 Fe 1.5	41 Al 0.45	0.9 Mn 0.15	—	870

Experiments on Rails. ANON. (*Amer. Mech.*, xxxvii, 3, 125.)—Just at present many experiments are being made to produce a rail that will be more resistant to wear than those now in use. According to the United States Geological Survey, over 150,000 tons of nickel and nickel-chromium steel rails were rolled in 1910, besides about 80,000 tons of chromium, manganese, vanadium, and other similar steels. Most of the experimenting, however, was done on titanium steels, 250,000 long tons of which were rolled. There need be no titanium left in the steel: its value lies in its absorptive power for nitrogen, oxygen, and other impurities.

Calcium Cyanamide Production. ANON. (*Oil, Paint and Drug Reporter*, July 29, 1912.)—The American Cyanamide Company has contracted for the extension of its calcium cyanamide plant at Niagara Falls, Ont., in addition to new machinery, at an approximate cost of $500,000, which will more than double the present capacity of the factory. This company uses the Frank and Caro process. Now the company uses 6000 electric horsepower. It began the manufacture of cyanamide in January, 1910; and during that year exported to the United States product valued at $211,202. In 1911 the value of the cyanamide shipped to the United States was $338,774, and, judging from exports during the present year, the increase will be much greater.

Progress of the Fixation of Nitrogen Process in Scandinavia. (*Times, Eng. Suppl.*, Oct. 15, 1913.)—The successful commercial development of the process for oxidizing ammonia to nitric acid by means of the catalytic action of platinum is described. The carbide works at Odda, Norway, when operations were started, comprised ten furnaces, each with a capacity of seven to eight tons of carbide per 24 hours. Ten more furnaces, each of 16 to 18 tons capacity, have recently been added. The temperature is maintained at 5720° F. (3160°C.), the electrodes of the older furnaces taking 1400 kilowatts and those of the newer ones 3500 kilowatts. It is stated that the radiation of heat from the new furnaces is less than from a steam boiler. The hot gases from the furnaces are used in the lime-kilns instead of producer gas. The annual output, 32,000 tons up to recently, is now 80,000 tons of carbide. Most of this carbide passes direct to the calcium cyanamide works of an associated company, which at the present time has an annual output of 80,000 tons, the number of furnaces having been increased nearly seven-fold since the works started four years ago. The nitrogen used is prepared by the Linde process, 100 tons of air being liquefied daily. The furnaces have a capacity of one ton each; the nitrogen is supplied through valves in the side, and a current at 60 to 75 volts through an electrode in the centre. After about 24 hours at 1650° F. (900° C.) the carbide is converted into cyanamide, the product containing 20 per cent. of nitrogen. In addition to these developments a company formed to acquire the rights of the Ostwald process (except in Westphalia and Rhineland) is erecting new works for the manufacture of carbide and cyanamide (and nitric acid) at Aura, Norway, with an estimated output of 200,000 tons of calcium cyanamide, and has acquired waterfalls at other places in Norway and also in Iceland, capable of furnishing power for an annual output of nearly 2,000,000 tons of calcium cyanamide. Works are to be erected at Dagenham on the Thames, at Trafford Park, Manchester, in Scotland, and in Ireland for producing from calcium cyanamide 12,000, 12,000, 9000, and 3000 tons of nitric acid respectively. In the event of the supply of calcium cyanamide being curtailed or stopped, it will be possible to use the ammoniacal liquors from coke ovens, blast furnaces, residual recovery plant, and gas works as sources of ammonia.

Spotting-Out and Silver Plating. C. F. BURGESS and L. T. RICHARDSON. (*Amer. Inst. of Metals*, Oct., 1913.)—The gradual formation of spots on plated articles, known as "spotting-out," is considered to be probably due to galvanic action. The surfaces of brass castings often contain minute pits which frequently enclose small quantities of electrolyte (cyanides), which form an electrical contact between the brass and silver. Since the electrolyte is not necessarily consumed in the process, but may serve simply as a conducting medium, it is possible for a very small amount to cause extensive corrosion.

Speed Regulation of Induction Motors. F. W. MEYER. (*Elektrotechn. Zeitschr.*, xxxiv, 903.)—This is a development of the theory of a machine for simultaneously regulating the speed and power factor of induction motors. The machine considered has an armature similar to that of a rotary converter, and is connected to the supply mains at the slip-ring end through the medium of a variable-ratio transformer, while at the commutator end it is connected to the slip-rings of the induction motor. The stator carries a light three-phase winding which is closed on an external adjustable resistance. The armature is not mechanically connected to any other machine. The torque required to overcome the friction and other rotational losses is furnished by the agency of the currents in the stationary secondary circuits, as in an ordinary induction motor, and the speed can be controlled by the rheostats in those circuits. The same control determines the frequency of the electromotive forces at the commutator, which are injected into the rotor-circuits of the main motor, and so fixes the speed of the latter. The correct relation between the magnitude and frequency of these electromotive forces which is necessary to produce a given speed and power-factor is obtained by simultaneous adjustment of the stator rheostat and the variable-ratio transformer which connects the armature to the supply.

Transformations of the Active Deposit of Thorium. E. MARSDEN and C. G. DARWIN. (*Proc. Roy. Soc.*, A87, 17.)—Experiments undertaken to discover the genetic arrangement of the various products of the active deposit of thorium, especially the nature of the transformations which take place in the product or products generally included under the name thorium C, have shown that of the atoms of thorium C, 35 per cent. emit α-particles of range 4.8 cm. and are converted into atoms of thorium D, while the remaining 65 per cent. emit β-particles and disintegrate into atoms of a very short-lived α-ray product, thorium C_2, range 8.6 cm. The transformations are regarded as of unusual interest, as the case furnishes undoubted evidence of atoms of a single kind of matter having two distinct modes of disintegration.

Tungsten Lamps Taking 0.5 Watt per Candle-Power. ANON. (*Elect. Rev. and West. Electn.*, lxiii, 156.)—The General Electric Company announces a new type of tungsten lamp operating at a specific consumption of only 0.5 watt per candle-power. Specially-shaped tungsten filaments in the form of loops of closely-wound spirals are used, and the bulb is filled with inert nitrogen at a pressure of about one atmosphere. It is anticipated that lamps of high candle-power taking six ampères and over will first be introduced, and that their higher efficiency will enable them to be used in fields not hitherto covered by incandescent lamps.

Einstein's Stationary Gravitation Field. P. EHRENFEST. (*Konink. Akad. Wetensch Amsterdam. Proc.*, xv, 1187.)—Discusses the hypothesis of equivalence on which Einstein bases his attempt at a theory of gravitation, and which requires a curvature of the rays of light in a field of attraction. The problem is put thus: Let a laboratory L, with the observers in it, have some accelerated motion with regard to a system of coördinate x, y, z, which is not accelerated. Let it, for example, move parallel to the z-axis with some positive acceleration. Then the observers will find that all the inert masses which are at rest with regard to the laboratory exert a pressure on the bodies which are in contact with their bottom sides. There are two ways for these observers to explain this pressure: (1) Our laboratory has an acceleration upwards, hence all inert masses press on the bodies under them; (2) our laboratory is at rest; but a field of force acts in it which pulls the masses down. Observations on the course of the rays of light seem to make it possible to decide between the ways (1) and (2), for, with regard to the system of coördinate x, y, and z, the light travels rectilinearly, but with regard to the accelerated laboratory *curvilinearly*. The conclusion is reached that all statical fields of attraction, except those of a very particular class, are in contradiction with Einstein's hypothesis of equivalence. For example, the statical field due to several centres of attraction which are stationary with respect to each other is not compatible with the hypothesis of equivalence.

Hardened Metal. ANON. (*Amer. Mach.*, xxxix, No. 16, 662.)—According to a French contemporary, a metal can be regarded as hardened if it is possible to soften it by annealing. The extent of the hardening may be expressed numerically by the ratio obtained by dividing the hardness number of the metal by its hardness number when fully annealed. Such hardness ratios were obtained for silver, aluminum, aluminum bronze, and copper bronze, and were found to express satisfactorily the extent of hammer hardening. The ratios were obtained usually with a ball 0.12 inch in diameter under a pressure of 66 pounds. The hardness ratios were about the same when 2200 pounds pressure was applied. By using the lesser pressure the hardness of bismuth was determined as 4.6 to 4.8 and of antimony as 58.42.

Electrification by X-Rays. C. G. BÉDREAG. (*Compt. Rendus*, clvii, 39.)—Experiments were made on the electrification of electrodes of brass and silver exposed at low pressures to X-rays. As the pressure is decreased below about 10^{-3} mm. mercury the electrodes become charged with an increasing positive charge, which is further increased when the electrodes are placed in a magnetic field. At atmospheric pressure the particular electrodes become negatively charged.

New Fuel. ANON. (*Amer. Mach.*, xxxix, No. 20, 834.)—A new fuel for automobiles and internal-combustion engines is announced, in British papers, under the name of Economin. This has for its base 80 per cent. of kerosene, the remainder being chemicals which form an emulsion. Then the mixture is distilled and gives a fuel which is cheaper than gasoline, in England at least, and which gives more power from the same motor. It is further claimed to give practically perfect combustion, so that there is almost no carbon deposit in the cylinders, to be almost odorless, and to make an easy-starting motor. It can be used with the same carburettor adjustment as gasoline. The fuel is not yet on the market, but a plant to turn out 20,000,000 gallons per year is under construction.

The Baldwin Locomotive Works. ANON. (*Amer. Mach.*, xxxix, No. 16, 652.)—This locomotive works, after 82 years of continuous operation, has produced 40,000 locomotives. The first was completed in 1832, when the industry was in its infancy. The five thousandth locomotive was built in 1850, eighteen years later, during which period two important types of freight locomotives, the "Mogul" and "Consolidation," were introduced and widely adopted. From 1889 to 1902, when locomotive number 20,000 was built, compound locomotives came into extensive use; and trailing wheels, allowing increased boiler capacity without imposing excessive loads on the driving wheels, were introduced. The average annual production for the five years from 1902 to 1907 was approximately 2000 locomotives, and locomotive No. 30,000 was built in 1907. During this period occurred the introduction of the Mallet articulated compound locomotive and the adoption of the Walschaerts valve motion. This period has been marked by the extensive use of superheaters, brick-arches, and other devices; and the mechanical stoker is another accessory developed during the past few years.

Nitrogen in Iron. W. HERWIG. (*Stahl und Eisen*, xxxiii, 1721.)—The gas contained in the blisters formed on steel plates during rolling and annealing was found to consist chiefly of nitrogen. The gases evolved during the solidification of iron immediately after tapping from the blast furnace include large quantities of hydrogen and carbon monoxide, white iron containing more hydrogen, and hot-blast gray iron more carbon monoxide. The nitrogen in steel turnings was reduced by heating in a current of hydrogen from 0.022 to 0.006 per cent., and, although it was not increased by heating in a current of nitrogen, yet from a mixture of nitrogen and hydrogen in equal proportions a steel was obtained, in one case, with as much as 0.052 per cent. nitrogen. The author considered that a similar action takes place in the converter. A steel containing 0.04 per cent. nitrogen, when tested, broke without elongation, but was improved by prolonged annealing.

Aluminum Bronze Containing Boron. ANON. (*Brass World*, ix, No. 12, 441.)—Edward D. Gleason, of New York, has recently patented a process for the use of boron in aluminum bronze. It is claimed that this alloy does not tarnish as readily as bronze without boron, and has superior physical properties. It is made in this way: Aluminum is fused in a crucible under a layer of a mixture of three parts fluorspar and 1 part of boric acid. A high heat is used, so that boron fluoride forms, which reacts on the aluminum and boron is reduced. This reduced boron is absorbed by the aluminum, forming an alloy. This alloy is used in the proportion of 10 parts of alloy to 90 parts of copper for making an aluminum bronze. The alloying is effected under a layer of charcoal and black oxide of manganese.

Removing Lead from Water. P. SCHMIDT. (*Chem. Zentr.*, ii, 1258.)—The lead contained in tap water is present as hydroxide or basic carbonate in the form of a suppresion or colloidal solution. On standing for a few days in tall cylinders the upper layers of the water are found to be richer in lead. In the case of water which has been boiled for a long time, the lower layers become richer in lead on standing. On the dialysis of water containing lead, no lead can be found in the outer water even in presence of excess of carbon dioxide. In accord with these facts showing that lead is present in the form of colloidal solution, it has been found that the lead can be completely removed by passing the water through a Berkefield filter or through a rapid filter of porous porcelain.

Influence of Drawing on the Properties of Metallurgical Products. L. GUILLET. (*Rev. de Métallurgie*, x, 769.)—This paper refers to the effect of cold work on the mechanical properties and the resistance of metals and alloys to corrosion. Drawing greatly increases the elastic limit and decreases the resilience. Elongation is diminished to a greater extent than the contraction. Experiments have been made on the resistance of aluminum and some of its alloys, to corrosion in water and in different saline solutions of various strengths. Although in a great many cases the cold worked product is attacked to a greater degree than the annealed product, this is by no means a general law. The action appears to be a function of the alloy and the acting reagent.

Aluminum Cooking Utensils. ANON. (*The Metal Industry*, ii, No. 12, 510.)—Prof. John Glaister, of Glasgow University, tested aluminum cooking utensils to ascertain if food was in any way injured by being prepared in them. The only substances that dissolved any of the metal were oranges, lemons, Brussels sprouts, and tomatoes, but even in these cases the quantity was so small as to be absolutely harmless.

Gripping Power of Portland Cement. H. LUFTSCHITZ. (*Tonind. Zeit.*, xxxvii, 1491.)—The setting of Portland cement is assumed to be due to the formation of a gel, which ultimately hardens to a lime-aluminate-silicate mass and forms a close-fitting network around any embedded inert material. With concrete, such a network surrounding angular gravel or sand would be stronger than one enclosing rounded grains, and the strength attained with washed sand would be greater than unwashed, since, in the latter case, the intervening layer of soft clay prevents the surrounding network from directly gripping the grains of sand. Similarly, with ferroconcrete, rust is detrimental as preventing close contact between the iron and the surrounding gel.

Scandium in American Wolframite. H. S. LUKENS. (*J. Amer. Chem. Soc.*, xxxv, 1470.)—Meyer has shown that wolframite residues from Zinnwald contained sufficient scandium to make them a productive source of this element. Following Meyer's methods, the author has isolated pure scandium oxide from American wolframite residues from Colorado. From 3.2 kilos. of the residues, about 1.6 grammes of crude scandium oxide were obtained.

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